THE
WORKS OF ARISTOTLE

TRANSLATED INTO ENGLISH
UNDER THE EDITORSHIP

OF

J. A. SMITH M.A.
FELLOW OF BALLIOL COLLEGE

W. D. ROSS M.A.
FELLOW OF ORIEL COLLEGE

PART 2
DE LINEIS INSECABILIBUS

OXFORD
AT THE CLARENDON PRESS

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GENERAL

PREFACE

It was the desire of the late Master of Balliol, Dr. Benjamin Jowett, as formulated in his will, that the proceeds from the sale of his works, the copyright in which he bequeathed to Balliol College, should be used to promote the study of Greek Literature, especially by the publication of new translations and editions of Greek authors. In a codicil to his will he expressed the hope that the translation of Aristotle's works begun by his own translation of the *Politics* should be proceeded with as speedily as possible. The College resolved that the funds thus accruing to them should, in memory of his services to the College and to Greek letters, be applied to the subvention of a series of translations of the works of Aristotle. Through the co-operation, financial and other, of the Delegates of the University Press it has now become possible to begin the realization of this design. By agreement between the College and the Delegates of the Press the present editors were appointed to superintend the carrying out of the scheme. The series, of which the first instalment is now brought before the public, is published at the joint expense and risk of the College and the Delegates of the Press.

The editors have secured the co-operation of various scholars in the task of translation. The translations make no claim to finality, but aim at being such as a scholar might construct in preparation for a critical edition and commentary. The translation will not presuppose any critical reconstitution of the text. Wherever new readings are proposed the fact will be indicated, but notes justificatory of conjectural emendations or defensive of novel interpretations will, where

PREFACE

admitted, be reduced to the smallest compass. The editors, while retaining a general right of revision and annotation, will leave the responsibility for each translation to its author, whose name will in all cases be given.

Translators have been found for the *Organon*, *Physics*, *De Caelo*, *De Anima*, *Historia Animalium*, *De Animalium Generatione*, *Metaphysics*, *Eudemian Ethics*, *Rhetoric*, and *Poetics*, and it is hoped that the series may in course of time include translations of all the extant works of Aristotle. The editors would be glad to hear of scholars who are willing to undertake the translation of such treatises as have not already been provided for, and invite communications to this end.

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J. A. S.
W. D. R.

</div>

March, 1908.

PARTS PUBLISHED

1. PARVA NATURALIA: by J. I. BEARE and G. R. T. ROSS.
2. DE LINEIS INSECABILIBUS: by H. H. JOACHIM.

DE LINEIS

INSECABILIBUS

BY

HAROLD H. JOACHIM

FELLOW OF MERTON COLLEGE

OXFORD

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GENERAL

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INTRODUCTORY NOTE

THE treatise Περὶ ἀτόμων γραμμῶν, as it is printed in Bekker's Text of Aristotle, is to a large extent unintelligible. But M. Hayduck, in the valuable paper which he contributed to the *Neue Jahrbücher für Philologie und Paedagogik* (vol. 109, part I, Teubner, 1874), prepared the way; and Otto Apelt, profiting by Hayduck's labours and by a fresh collation of the manuscripts, published a more satisfactory text in his volume *Aristotelis quae feruntur de Plantis*, &c. (Teubner, 1888). Many of the most difficult passages are discussed and elucidated in the *prolegomena* to this volume. Finally, Apelt included a German translation of the treatise in his *Beiträge zur Geschichte der griechischen Philosophie* (Teubner, 1891).

In the following paraphrase, I have endeavoured to make a full use of the work of Hayduck and Apelt, with a view to reproducing the subtle and somewhat intricate thought of the author, whoever he may have been. Though the treatise is published amongst the works of Aristotle, there are grounds for ascribing it to Theophrastus: whilst, for all we can tell, it may have been written neither by Aristotle nor by Theophrastus, but by Strato, or possibly by some one otherwise unknown. But the work—no matter who wrote it—is interesting for the close texture of its reasoning, and for the light which it throws on certain obscure places in Plato and Aristotle. Its value for the student of the History of Mathematics is no doubt considerable: but my own ignorance of this subject makes me hesitate to express an opinion.

I take this opportunity of thanking three of my friends, E. I. Carlyle (Fellow of Lincoln College) and A. L. Dixon (Fellow of Merton College) for their help in several of the mathematical passages, and W. D. Ross (Fellow of Oriel College) for his valuable suggestions, most of which I have adopted.

<div align="right">H. H. J.</div>

January, 1908.
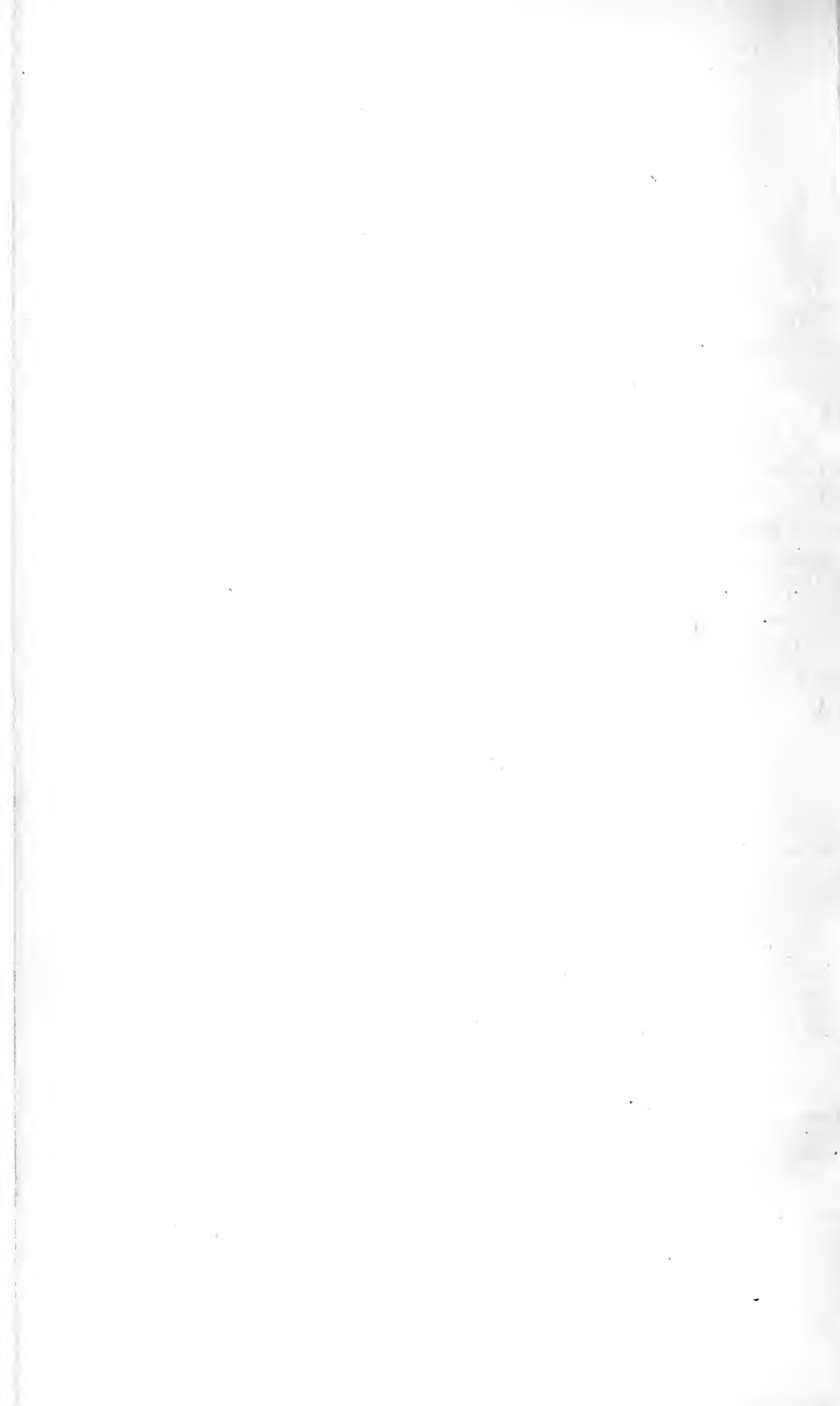
ARE there indivisible lines? And, generally, is there a simple unit in every class of quanta?[1]

§ 1. Some people maintain this thesis on the following grounds :—

(i) If we recognize the validity of the predicates 'big' and 'great', we must equally recognize the validity of their opposites, 'little' and 'small'. Now that which admits practically an infinite number of divisions, is 'big' not 'little' ⟨or 'great' not 'small'⟩.[2] Hence, the 'little' quantum and the 'small' quantum will clearly admit only a finite number of divisions.[3] But if the divisions are finite in number, there must be a simple magnitude. Hence in all classes of quanta there will be found a simple unit, since in all of them the predicates 'little' and 'small' apply.

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[1] ᵃ1. ἐν ἅπασι τοῖς ποσοῖς, and ᵃ8 ἐν ἅπασιν. The theory maintains that in dividing any quantum, of whatever kind, you will ultimately come to indivisible constituent quanta of the same kind. Every line, e.g., is composed of a finite number of indivisible lines : every solid of a finite number of indivisible solid constituents, i.e. solids not further divisible into solids. The advocates of this theory were feeling after the conception on which the differential calculus was based, and I presume that in the history of Mathematics they would take their place as the forerunners of Newton and Leibniz. Cf. Hegel, *Wissenschaft der Logik*, vol. i. pp. 302-4.

ᵃ2. ἐστί τι ἀμερές. I translate ἀμερές throughout by 'simple', using 'simple'—in opposition to 'complex' or 'composite'—as equivalent to 'without parts'.

[2] ᵃ4 ff. τὸ πολύ and τὸ ὀλίγον—that which contains many, and that which contains few, units—are the opposite predicates of discrete quanta, i.e. of Number (cf. Arist. *Met.* 992ᵃ 16, 17) : τὸ μέγα and τὸ μικρόν apply to continuous quanta. This at least seems to hold of the primary signification of these terms ; but the distinction is not maintained. Thus, e.g., in the *Categ.* 4ᵇ 20 ff., Number is instanced as a discrete quantum, Time and Surface are quoted *inter alia* as continuous quanta ; but πολύς is predicated of Surface (5ᵇ 2), and of Time (5ᵇ 3). I have added ⟨or 'great' not 'small'⟩ in my translation, to complete the writer's thought. I do not suggest that there is an omission in the text.

[3] ᵃ7. I translate ἔχει διαιρέσεις throughout as 'admits divisions', though at times the meaning of the Greek passes into 'contains divisions': cf., e.g., 969ᵃ 8.

9   (ii) Again, if there is an Idea of line, and if the Idea is first of the things called by its name [1] :—then, since the parts are by nature prior to their whole, the Ideal Line must be indivisible.[2] And, on the same principle, the Ideal Square, the Ideal Triangle, and all the other Ideal Figures—and, generalizing, the Ideal Plane and the Ideal Solid—must be without parts: for otherwise it will result that there are elements prior to each of them.

14   (iii) Again, if Body consists of elements,[3] and if there is nothing prior to the elements, Fire and, generally, each of the elements which are the constituents of Body must be indivisible: for the parts are prior to their whole. Hence there must be a simple unit in the objects of sense as well as in the objects of thought.[4]

---

[1] ᵃ9, 10. ἡ δ' ἰδέα πρώτη τῶν συνωνύμων, i.e. the Idea is conceived as the limiting member of a series of things called by the same name and sharing the same nature in various degrees. Thus all lines, *quâ* participating in the same linear nature, are called by the same name, 'line.' The Idea of Line is the Ideal Line which exhibits this linear nature perfectly and precisely: it is the limit from which actual lines derive, or to which they more or less approximate. If all lines were arranged in a series according to the degrees in which linearity obtained expression in them, the Idea of Line would be the first member of the series: it would be the Ideal Line which was *just* 'Line', neither more nor less.

[2] ᵃ11. I accept Hayduck's conjecture ἀδιαίρετος, for the MSS. διαιρετή, of which I can make nothing.

The theory contemplated by this argument is that in every kind of quantum—and, within spatial quanta, in every type of plane and of solid figure—there is an Ideal Quantum in the sense explained in the preceding note. This Ideal Quantum, it is argued, must be 'indivisible', i.e. simple. For, *quâ* Ideal, it is the primary member in the series of which it is the Idea; but, if it had parts, they would be prior to it, since the parts are prior to their whole.

[3] ᵃ14. ἔτι εἰ σώματός ἐστι στοιχεῖα . . . Bekker. Read ἔτι εἰ σώματος ἔστι στοιχεῖα, 'if there are elements of Body.' (The variant σώματα, though well attested, does not seem right.) σῶμα here, as the context shows, is not (as in l. 13) mathematical solid, but perceptible or physical body.

[4] The first two arguments were directed to show that simple units are involved (i) in the Quanta of Mathematics, and (ii) in the Ideal Quanta postulated by a certain metaphysical theory. The present argument is intended to prove that the perceptible bodies (the bodies of Physics and of everyday life) ultimately consist of simple constituents. According to current views, all material things—all αἰσθητὰ σώματα—consisted in the end of certain elementary constituents, viz. Earth, Air, Fire, and Water. An 'Element' means what is primordial, and therefore (it is argued) it must be without parts.

The writer does not explain to what precise form of physical theory he is alluding. He seems to be thinking of the somewhat vague and

(iv) Again, Zeno's argument proves that there must be 18 simple magnitudes.[1] For the body, which is moving along a line, must reach the half-way point before it reaches the end. And since there always is a half-way point in any 'stretch' which is not simple, motion—unless there be simple magnitudes—involves that the moving body touches succes- 20 sively one-by-one an infinite number of points in a finite time: which is impossible.[2]

But even if the body, which is moving along the line, does touch the infinity of points in a finite time, an absurdity results. For since the quicker the movement of the moving body, the greater the 'stretch' which it traverses in an equal time: and since the movement of thought is quickest of all 25 movements:—it follows that thought too will come succes- sively into contact with an infinity of objects in a finite time. 968ᵇ And since 'thought's coming into contact with objects one- by-one' is counting, we must admit that it is possible to count the units of an infinite sum in a finite time. But since this is impossible, there must be such a thing as an 'indivisible line'.[3]

popular view, which regarded Earth, Air, Fire, and Water as the 'Letters' of the Alphabet of Reality, and the physical universe as a complex of 'Syllables' and 'Words' in which these four Letters are variously com- bined. But the *principle* of the argument would apply to the more refined forms which the theory assumes in the *Timaeus* of Plato and in Aristotle's physical writings. The primordial triangles of the *Timaeus*, *quâ* Elements of all bodies, are presumably without physical parts, i.e. physically indivisible. And the Earth, Air, Fire, and Water, which (according to Aristotle) are the chemical constituents of all ὁμοιομερῆ—and therefore the primary constituents of all composite bodies—, are 'τὰ ἁπλᾶ σώματα', although the character of each of them is dual, i.e. is exhibited in two of the four fundamental qualities. (For Aristotle's theory of the Elements, cf. my article on 'Aristotle's Conception of Chemical Combination', *Journal of Philology*, No. 57.)

[1] ᵃ19. ἀνάγκη τι μέγεθος ἀμερὲς εἶναι, i.e. there must be *such a thing as* a simple magnitude. For Zeno's argument cf. Arist. *Phys.* 187ᵃ 1 and Simplicius *ad loc.*

[2] ᵃ18–23. Here and elsewhere I have not scrupled to paraphrase rather freely, in order to bring out the argument. From the infinite divisibility of the continuous path of the moving body, Zeno concluded that motion was impossible; for the moving body would have to come successively into contact with an infinite number of points in a finite time. The advocates of 'simple units' argue that, *since motion is a fact*, the con- tinuous path cannot be divisible *ad infinitum*: i.e. any given line must consist of a finite number of 'indivisible lines'.

[3] ᵇ4. The Greek is εἴη ἂν τις ἄτομος γραμμή. The meaning here (as in

4   (v) Again, the being of 'indivisible lines' (it is maintained) follows from the Mathematicians' own statements. For if we accept their definition of 'commensurate' lines as those which are measured by the same unit of measurement,[1] and if we suppose that all commensurate lines actually are being measured,[2] there will be some actual length, by which all of them will be measured.[3] And this length must be indivisible. For if it is divisible, its parts—since they are commensurate with the whole—will involve some unit of measurement

10 measuring both them and their whole. And thus the original

968ᵇ 5 : cf. also 968ᵃ 19) cannot be given by the English 'there must be an indivisible line' or 'a line which is indivisible'. We must translate either as above, or by the plural 'there must be indivisible lines'.

The argument (ᵃ23–ᵇ4) is directed against a particular view of thought and of counting. 'Assume'—the writer says in effect—'that the moving body does in fact touch an infinity of points one-by-one in a finite time. According to your view that thought is the quickest of all movements, it will follow *a fortiori* that thought touches an infinity of objects one-by-one in a finite time: i.e. (according to your definition of counting) that we can count an infinite number in a finite time. But this is impossible. And the only way to avoid this absurdity, *whilst recognizing the fact of motion*, is to postulate "indivisible lines".'

The theory that thinking is a movement of the Soul was not held by Aristotle: for he argues in the *de Anima* (A. ch. 3) against all attempts to define the Soul as 'that which moves itself,' and maintains that 'it is impossible that movement should be a property of the Soul' (l. c. 406ᵃ 2 ff.). Certain speculations of Plato in the *Timaeus* (which Aristotle criticizes, l. c. 406ᵇ 26 ff.) regard thought as a movement: and Theophrastus and his pupil, Strato, are known to have maintained that thought was a movement of the Soul (cf. Apelt, *Beiträge* &c., p. 270). But we must not infer—as Apelt (l. c.) does—that Aristotle is not the author of the present treatise: still less that it was written by Theophrastus or Strato. For we are here dealing with an *argumentum ad hominem*, and the writer is not himself committed to the view that thought is a movement of the Soul.

[1] Cf. Euclid, *Elements*, Bk. X, def. i Σύμμετρα μεγέθη λέγεται τὰ τῷ αὐτῷ μέτρῳ μετρούμενα, ...

[2] ᵇ6, 7, reading (with all the MSS., except N) εἰ σύμμετροί εἰσιν αἱ τῷ αὐτῷ μέτρῳ μετρούμεναι, ὅσαι δ' εἰσὶ σύμμετροι, πᾶσαί εἰσι μετρούμεναι.

Apelt *in his text* followed N, and read ὅσαι δ' εἰσὶ μετρούμεναι, πᾶσαί εἰσι σύμμετροι. But *in his translation* he reverts to the best attested reading.

I substitute a comma for Bekker's colon after μετρούμεναι in l. 6, because the whole clause is dependent on εἰ. The logic of the passage is, 'If we accept *x*, and combine with that the supposition *y*, there must be indivisible lines: for *on those suppositions* there will be a unit length which must be indivisible.'

[3] ᵇ8. ᾧ πᾶσαι μετρηθήσονται, 'whereby all *commensurate* lines will be measured': but, as appears from 969ᵇ 10–12, the argument (by a somewhat transparent fallacy) regarded *all lines* as 'commensurate'. See next note.

unit of measurement would turn out to be twice one of its parts, viz. twice its half.[1] But since this is impossible, there must be an indivisible unit of measurement.[2] And just as all the lines, which are compounded of the unit, are composed of 'simples', so also the lines, which the unit measures once, consist of 'simples'.[3]

And the same can be shown to follow in the plane figures too. For all the squares, which are drawn on the rational 15 lines, are commensurate with one another; and therefore ⟨by the preceding argument⟩ their unit of measurement will be simple.[4]

[1] ᵇ10, 11. Bekker reads ὥστε μέρους τινὸς εἴη [εἶναι Wᵃ] διπλασία [διπλασίαν rc N, διπλάσιον LWᵃ] τὴν ἡμίσειαν, .•. . From the reading of LWᵃ, I suspect that the author wrote διπλασίων (cf. e. g. Euclid, *Elements*, Bk. X, prop. 9: the word occurs in [Arist.] *Probl.* 923ᵃ 3, *De Mundo*, 399ᵃ 9). In place of τὴν ἡμίσειαν, Zᵃ apparently (' ut videtur ', Apelt says in his *apparatus criticus*) reads τῆς ἡμίσυος. Hayduck conjectured ὥστε μέτρον ἂν εἴη διπλασία τῆς ἡμισείας, or ὥστε μετρεῖν ἂν εἴη διπλασίᾳ τὴν ἡμίσειαν. Apelt suggests ὥστε μέρους τινὸς ἂν εἶναι διπλασίαν τὴν ἡμίσειαν, but I do not see that this is of much assistance. I have translated as if the text were ὥστε μέρους τινὸς ⟨ἂν⟩ εἴη διπλασίων, τῆς ἡμισέας· ἐπεὶ δὲ κτλ. But it is possible that τῆς ἡμισέας ought to be excised as a gloss explanatory of μέρους τινός.

It appears (from the criticism of this argument at 969ᵇ 10-12) that the advocates of 'indivisible lines' reasoned thus:—' Lines measured by the same unit are "commensurate". Now take any line, *AB*. It will always be possible to find, or draw, a line containing without remainder a multiple of the units in *AB*: i. e. *AB* will be "commensurate". Let then all "commensurate" lines (i. e. *all lines*) be actually measured. There will be an actual length, or infinitesimal line, *xy*, which is the unit of measurement of them all. And *xy* must be indivisible. For, if not, *xy* will have parts: and thus the unit will be multiple (v. g. will be twice its own half), which is absurd.' The fallacy is obvious, and is exposed at 969ᵇ 10-12. Any line *AB* can become ' commensurate' with *some* line: but, because commensurate with *some* line, it is not necessarily commensurate with *all* lines, or ' commensurate' absolutely. One would indeed think the fallacy too obvious to have been committed: but, in the refutation, the writer refers to it as a ridiculous and obvious sophism, cf. 969ᵇ 6-10 and 12-15.

[2] ᵇ11. The MSS. read ἡμίσειαν, ἐπειδὴ τοῦτ' ἀδύνατον ἂν εἴη μέτρον.
I read with Apelt ἐπεὶ δὲ τοῦτ' ἀδύνατον, ⟨ἀδιαίρετον⟩ ἂν εἴη μέτρον, and place a colon before ἐπεί. The insertion of ἀδιαίρετον was suggested by Hayduck, after the Latin translator, Julius Martianus Rota, who writes ' quoniam vero hoc fieri nequit, indivisibilis esse mensura debet'.

[3] ᵇ12-14. Let *xy* be the unit of measurement, which measures all commensurate (i. e. *all*) lines. Then all lines will ' consist' of simples: for they will either contain *xy* once, or more than once, without remainder.

[4] ᵇ14-16. The object of this argument is to show that 'simple units' must be admitted in plane figures, as well as in lines. The writer selects the square as an example of plane figure, and maintains that all squares

16   But if ⟨*per impossibile*⟩ any such unit-square be cut along
any prescribèd and determinate line, that line will be neither
' rational ' nor ' irrational ', nor any of the recognized kinds of
⟨irrational⟩ lines which produce rational squares, such as the
'apotome' or the 'line ex duobus nominibus'.   Such lines,
20 at which the unit-square might be divided, will have no nature
of their own at all ; though, relatively to one another, they
will be rational or irrational.[1]

consist ultimately of a finite number of minimal squares, not themselves
divisible into any smaller plane figures.

In order to understand the argument, and the fallacy on which it rests,
it will be necessary to explain certain technical terms of Greek geometry.
(1) The expression τὰ ἀπὸ τῶν ῥητῶν γραμμῶν (l. 15) must—in accordance
with Euclid's invariable usage—mean ' the squares on the ῥηταὶ γραμμαί '.
The noun implied is τετράγωνα : but τὸ ἀπό followed by the genitive is
constantly used without τετράγωνον, and always means the square on such-
and-such a line.   (Hence Apelt is wrong in translating ' Alle *Flächen*
mit rationalen Seitenlinien '.)   (2) The proper meaning of ῥηταὶ γραμμαί
will be seen from the following definitions of Euclid (*Elem.* X) :—def. 3 ' . . .
given any straight line, there are an infinity of straight lines commen-
surate with it and an infinity incommensurate with it—incommensurate
either in length only, *or* both in length and in respect to the areas which
they and it produce if squared (αἱ μὲν μήκει μόνον, αἱ δὲ καὶ δυνάμει: see below).
Let the given straight line, and all the straight lines which are commen-
surate with it (whether commensurate both μήκει and δυνάμει, or δυνάμει only),
be called " Rational " (ῥηταί) : and let the straight lines, which are incom-
mensurate with it, be called " Irrational " (ἄλογοι) ' : def. 4 ' And let the
square on the given straight line, and all the squares commensurate
therewith, be called " Rational " : and let the squares incommensurate
with it be called " Irrational " . . .'   (3) Any straight lines, which are
multiples of the same unit of length, are said to be σύμμετροι μήκει.   If e.g.
the unit of measurement be ἡ ποδιαία (the line one foot long), all lines con-
taining a whole number of feet are σύμμετροι μήκει.   But lines which do not
contain a whole number of the same unit of length are said to be σύμμετροι
δυνάμει, if they form squares containing a whole number of the same unit
of area.   All lines, which are σύμμετροι μήκει, are necessarily also σύμμετροι
δυνάμει—but the converse does not hold (Eucl. *Elem.* X, prop. 9, Coroll.).

We are now in a position to understand the argument of ᵇ14–16.   The
writer extends the relative term ' rational ' illegitimately (making it
absolute), just as before he illegitimately extended the relative term
' commensurate '.   All ' rational ' lines are by definition δυνάμει σύμμετροι :
and therefore all squares on rational lines are commensurate.   And if we
suppose them actually measured, there will be an actual minimal square,
the unit of measurement of them all (cf. above, 968ᵇ 6–8) : and this
minimal square can be shown to be indivisible—i.e. not to contain smaller
plane figures—as before the unit-line was shown to be ἀδιαίρετον (968ᵇ 8–12).
But—*unless we assume that all lines consist of indivisible and equal unit-
lines*—we cannot assume that *all* lines are ' rational ' in Euclid's sense,
nor that *all* squares are commensurate with one another.

[1]   ᵇ16–21. The text of this passage is corrupt, and the argument obscure,
and I have no confidence in the interpretation which I have given.   As

§ 2. To these arguments we must make the following 21
answers :—

(i) (a) In the first place, it does not follow that the quantum,
which admits an infinite number of divisions, is not 'small' or
'little'.   For we apply the predicate 'small' to place and
magnitude, and generally to the continuous (and in some quanta
the predicate 'little' is suitably applied)[1] ; and nevertheless

regards the text, I adopt Apelt's reading in l. 19, ὧν δυνάμεις ῥηταί, οἶον
ἀποτομὴ ἢ ἡ ἐκ δυοῖν ὀνομάτοιν for the MSS. ὧν δὴ νῦν [νῦν δή N] εἴρηται,
οἶον ἀποτομὴν ἐκ δυοῖν ὀνομάτοιν.

The lines called ἐκ δυοῖν ὀνομάτοιν and ἀποτομή are two types of Irrationals
(i.e. μήκει ἀσύμμετροι, but δυνάμει σύμμετροι) which play a large part in
Euclid, *Elem.* Bk. X.

The line ἐκ δυοῖν ὀνομάτοιν is defined in Prop. 36 thus : — 'If two rational
straight lines, which are commensurate δυνάμει only, be added together, the
whole line is irrational : let it be called
"the line ἐκ δύο ὀνομάτων"' :—i.e. the
line AC is that type of 'Irrational'
(irrational relatively to AB and BC) which is called 'ex duobus nomi-
nibus', if it is such, that AB^2 is commensurate with BC^2, but AB
is incommensurate with BC. AB and BC are called the 'ὀνόματα'
of AC.

The ἀποτομή is defined in Prop. 73 thus :—' If from a rational line there
be taken a rational line commensurate with the whole line δυνάμει only,
the remainder is irrational : let it be called an "ἀποτομή"' :—i.e. if the
line AB be divided at C, so that AB^2
is commensurate with CB^2 but AB is
incommensurate (μήκει) with CB, then
AC is called an ἀποτομή.   The complementary part of the whole line
(viz. CB) is called relatively to AC its προσαρμόζουσα (cf. Propp. 79–84).
We might illustrate these two types of 'Irrationals' thus :—(1) Let the
two ὀνόματα be 1 and $\sqrt{5}$.   Then the whole line, $AB + BC$, $= (1 + \sqrt{5})$.
1 is incommensurate with $\sqrt{5}$, but (1)² and $(\sqrt{5})^2$ are commensurate.
(2) Let the whole line, AB, be $\sqrt{5}$.   Divide AB at C, so that $CB = 1$.
Then $(\sqrt{5})^2$ is commensurate with (1)², but $\sqrt{5}$ is incommensurate with 1.
AC (the ἀποτομή) $= (\sqrt{5} - 1)$.

I have interpreted the argument (968ᵇ 16–21) as a *reductio ad absurdum*.
'Suppose,' the writer urges, 'the unit-square is divided.   The line dividing
it will not answer to any known line : i.e. there is no line recognized by
Geometry at which the unit-square could be divided into smaller plane
figures.   For whatever line of division be selected, that line will neither
be rational nor irrational : nor will it fall under any of the recognized types
of line which, though irrational *quâ* lines, produce rational squares, or
otherwise exhibit relations studied by Geometry.   Any such lines of
division will, in fact, belong to a new order of lines, which may be
expressed as rational or irrational in terms of one another, but not in
terms of the ordinary geometry of lines.'

[1] ᵇ24. καὶ ἐφ' ὧν μὲν ἁρμόττει τὸ ὀλίγον . . .
Hayduck suggested καὶ ἐφ' ὧν ἁρμόττει, ὀλίγον, which would be an
improvement, though the excision of μέν seems unnecessary.   (It is,

we affirm that these quanta admit an infinite number of divisions.

25   (i) (b) Moreover, if in the composite magnitude there are contained ⟨indivisible⟩ lines,[1] the predicate ' small ' is applied to these indivisible lines, and each of them contains an infinite

969[a] number of points.   But each of them, *quâ* line, admits of division at a point, and equally at any and every point : hence each of these indivisible lines would admit an infinite number of divisions just like the non-indivisible lines.[2]   Moreover, some amongst the non-indivisible lines are ' small '.   But every non-indivisible line admits of division in accordance with any prescribed ratio : and the ratios, ⟨in accordance with which any such line may be divided⟩, are infinite in number.[3]

however, omitted by Z[a].)   Apelt defends the MSS. reading, but interprets καὶ ἐφ' ὧν—ὀλίγον as part of the subject of the sentence, taking μικρόν as predicate of the whole.   This seems difficult, because (*a*) the μὲν [ἐφ' ὧν μὲν] is purely gratuitous, and (*b*) there is no reason why the writer should over-ride the distinction between μικρόν and ὀλίγον.

If the τό be retained, the clause must, I think, be treated as parenthetical and interpreted as above.

[1] [b]25. Apelt reads (with N) ἔτι δ' εἰ ἐν τοῖς συμμέτροις γραμμαί εἰσι γραμμαί.   He suggests that the passage ought to be emended to run ἔτι δ' εἰ ἐνίαις σύμμετροι γραμμαῖς εἰσὶ γραμμαί, κατὰ τούτων ἄτομον λέγεται τὸ μικρόν.   Of this I can make nothing : nor do I see how he could defend his translation 'und von ihrem Mass gesagt wird, es sei unteilbar klein '.

All the MSS. (except N and Z[a]) read ἔτι δ' εἰ ἐν τῷ συνθέτῳ γραμμαί, κατὰ τούτων τῶν [τῶν omit t. NPW[a]Z[a]] ἀτόμων κτλ.   But as in PW[a] and L there is a lacuna after συνθέτῳ, I have ventured to conjecture ἔτι δ' εἰ ἐν τῷ συνθέτῳ ⟨ἄτομοί εἰσι⟩ γραμμαί . . .   With συνθέτῳ I understand μεγέθει or μήκει.

[2] [a]2, 3. The text in Bekker is καὶ ὁμοίως καθ' ὁποιανοῦν ἀπείρους ἂν ἔχοι διαιρέσεις ἅπασα ἡ μὴ ἄτομος.   I follow Apelt in placing a colon after ὁποιανοῦν, and in reading ἀπείρους οὖν ἔχοι . . .   After ἅπασα I insert ἄν, ὡς, combining the readings of NZ[a] and H[a].   The passage then runs καθ' ὁποιανοῦν· ἀπείρους οὖν ἔχοι διαιρέσεις ἅπασα ἄν, ὡς ἡ μὴ ἄτομος.

[3] [a]3–5. The text given in Bekker is ἔνιαι δὲ τούτων εἰς μακρὰ [μικρὰ LZ[a], μικρὰν NH[a]] καὶ ἄπειροι οἱ λόγοι [for οἱ λόγοι LPH[a]W[a] read ὀλίγον, Z[a] reads καὶ ὀλίγον].   πᾶσαν δὲ τμηθῆναι τὸν ἐπιταχθέντα δυνατὸν τὴν μὴ ἄτομον.   [For τὴν μή H[a] reads τομὴν τήν, and N places τὸν ἐπιταχθέντα after ἄτομον.]

I have ventured to read ἔνιαι δὲ τούτων εἰσὶ μικραί· πᾶσαν δὲ τμηθῆναι δυνατὸν τὴν μὴ ἄτομον τὸν [? κατὰ τὸν] ἐπιταχθέντα λόγον· καὶ ἄπειροι οἱ λόγοι. If this be thought too bold, we might retain the MSS. order, and read ἔνιαι . . . μικραί· καὶ ἄπειροι οἱ λόγοι, πᾶσαν . . . τὴν μὴ ἄτομον τὸν ἐπιταχθέντα. We must then take καὶ ἄπειροι οἱ λόγοι closely with the following words. The only authority for λόγον (which Apelt inserts after ἐπιταχθέντα) is the *editio princeps*.

(i) (c) Again, since the 'great' is compounded of certain ₅
'smalls', the 'great' will either be nothing, or it will be
identical with that which admits a finite number of divisions.[1]
For the whole admits the divisions admitted by its parts: i.e.
its divisions are finite or infinite, according as *their* divisions
are finite or infinite.[2] It is unreasonable that, whilst the small
admits a finite number of divisions only, the great should
admit an infinite number; and yet this is what the advocates
of the theory postulate.[3]

It is clear, therefore, that it is not *quâ* admitting a finite and ₁₀
an infinite number of divisions that quanta are called 'small'
and 'great' respectively.   And to argue that, because *in
numbers* the 'little' number admits a finite number of divi-
sions, therefore *in lines* the 'small' line must admit only a
finite number of divisions, is childish. For in numbers the
more complex are developed out of 'simples', and there is
a determinate something from which the whole series of the
numbers starts, and every number which is not infinite admits ₁₅

The argument of the whole passage (968ᵇ 25–969ᵃ 5) I take to be
as follows:—'Every composite length contains lines.   According to
the theory, some amongst these lines are "indivisible".   But every
one of these lines, *quâ* line, contains an infinity of points, and admits
therefore an infinity of divisions: for a point is that at which
a line can be divided.   Yet by comparison with the whole (com-
posite) length, all the "indivisible" lines, and at least *some* of the
divisible lines, are "small".   Hence infinitely-divisible quanta may be
"small".'

The λόγοι (969ᵃ 4) are, I presume, the numerical ratios in which any
line may be divided.

[1] ᵃ7. I accept Apelt's conjecture τὸ μέγα for the MSS. οὐ μέγα.

[2] ᵃ8. τὸ γὰρ ὅλον τὰς τῶν μερῶν ἔχει διαιρέσεις ὁμοίως, i. e. the divisions
which the whole admits—since it is the sum of its parts—are the sum of
the divisions which the parts admit, and the number of divisions is either
finite or infinite *in both cases*.   The argument, to which this is a reply,
assumed that the large number of divisions in the 'great' was 'practically
infinite' (968ᵃ 4), whilst the 'small' admitted only a finite number of
divisions.

[3] Reading, with Apelt, ἄλογον (for the MSS. εὔλογον) in ᵃ8, and οὕτω δ'
ἀξιοῦσιν (for the MSS. οὕτως ἀξιοῦσιν) in ᵃ10.

It is just possible, however, to retain the MSS. reading, if we con-
strue ἀξιοῦσιν as dative plural of the participle, and remove the stop
before οὕτως.   'And yet it is a reasonable inference for them, with
their assumptions, that the "small" admits a finite number, and the
"great" an infinite number, of divisions':—i. e. the view in question
has just been shown to be false, but it follows plausibly enough from
their premisses.

a finite number of divisions ; but in magnitudes the case is not parallel.[1]

17　(ii) As to those who try to establish the being of the indivisible lines by arguments drawn from the Ideal Lines, we may perhaps say that, in positing Ideas of these quanta, they are assuming a premiss too narrow to carry their conclusion ; and, by arguing thus, they in a sense destroy the premisses which they use to prove their conclusion. For their arguments destroy the Ideas.[2]

21　(iii) Again, as to the corporeal elements,[3] it is childish to postulate them as ' simple '. For even though some physicists do as a matter of fact make this statement about them, yet to assume this for the present inquiry[4] is a *petitio principii*. Or rather, the more obviously the argument would appear to involve a *petitio principii*, the more the opinion is confirmed that Solids and Lengths[5] are divisible in bulk and distance.[6]

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[1] The above arguments, from 968[b] 21, are directed against the first argument (968[a] 2–9) of the advocates of indivisible lines.

[2] [a]17–21. This is directed against the second argument (968[a] 9–14) of the advocates of indivisible lines.

κατασκευάζω is used in the sense of ' establishing ' (e. g. a conclusion or a definition) in opposition to ἀνασκευάζω, 'to overturn': cf. e. g. *Pr. Anal.* 43[a] 1, *Top.* 102[a] 15, &c. The argument in question aimed at proving the universal affirmative that *all* lines contain indivisible lines as ultimate constituents. And it tried to base this conclusion on the indivisibility of the Idea of line, i. e. it involved the assumption of Ideas of quanta, or at least of Ideas of lines. But from what holds good of Ideal lines, you can make no valid inference to *all* lines : the premiss is particular (Ideal Lines, i. e. *some* lines, are indivisible), and cannot serve as the basis of the universal conclusion which is to be proved.

Moreover, it is dangerous for the advocates of Ideas to use an argument of this kind. For their opponents may retort that, if the assumption of Ideal quanta leads to the absurdity of indivisible lines, then *so much the worse* for the Ideal theory. In the sphere of mathematics, they may say, the assumption leads to consequences mathematically absurd ; hence the whole theory of Ideas is discredited.

[3] [a]21. πάλιν δὲ τῶν σωματικῶν στοιχείων . . . The genitive alone seems impossible. I read πάλιν δ' ἐπὶ τῶν κτλ. (coll. 969[b] 6).

[4] [a]23. πρός γε τὴν ὑποκειμένην σκέψιν . . . I can find no exact parallel to this use of ὑποκειμένην, but cf. perhaps *Pol.* 1331[b] 36. In the next two lines ὅσῳ μᾶλλον . . . τόσῳ μᾶλλον is an expression without parallel in Aristotle.

[5] [a]26. Reading σῶμα καὶ μῆκος, and interpreting σῶμα as ' geometrical solid ' (not as ' perceptible body '). The difficulty in this reading is that καὶ τοῖς ὄγκοις καὶ τοῖς διαστήμασιν ought to mean ' both in bulk and distance ': but this would be true of σῶμα only. *Disjunctively*, of course, it is true of σῶμα and μῆκος, but the double καί is certainly awkward. Apelt in his translation adopts the reading of LNH[a]W[a] σῶμα μήκους : but he

(iv) The argument of Zeno does not establish that the [26] moving body comes into contact with the infinite number of points in a finite time, if the period and the path of the motion are considered on the same principle.[1]   For the time and the length are called ⟨both⟩ infinite and finite ⟨from different points of view⟩, and admit of the same divisions ⟨if considered both on the same principle⟩.[2]   [30]

Nor is ' thought's coming into contact with the members of an infinite series one-by-one' *counting*, even if it were supposed that thought does ' come into contact ' in this way with the members of an infinite series.   Such a supposition perhaps assumes what is impossible: for the movement of thought does not, like the movement of moving bodies, essentially involve *continua* and *substrata*.   969[b]

If, however, the possibility of thought moving in this fashion be admitted, still this moving is not ' counting ' ; for counting is movement combined with pausing.

It is absurd—we may perhaps suggest to our opponents—

can only translate this by making the μᾶλλον of l. 25 do double duty.   All would be plain if we could omit καὶ μῆκος altogether, and read σῶμα [i. e. ' perceptible body '] καὶ τ. ὄγκοις κ. τ. διαστήμασιν.

[6] [a]21-26. This paragraph is directed against the third argument (968[a] 14-18) of the advocates of indivisible lines.   That argument rested on the assumption that perceptible bodies involved Elements, i. e. *primary* constituents.   Even admitting that some physicists speak in this way about the constituents of bodies, to take this as a premiss to prove that there are indivisible magnitudes is to beg the question.   (Cf. Hayduck, l. c., p. 163, for the above interpretation.)   Or at least it looks like begging the question: and the more it looks so, the more the prevailing opposite opinion is confirmed.   For a view gathers strength in proportion to the weakness of the arguments advanced against it.

[1] [a]27, 28. The MSS. read . . . . συμβιβάζει οὐ συμπεπερασμένῳ χρόνῳ τῶν ἀπείρων ἅπτεται [LNPW[a]Z[a] : ἅπτεσθαι *ceteri*] τὸ φερόμενον ὡδὶ τὸν αὐτὸν τρόπον.

Bonitz conjectured τὸ ἐν πεπερασμένῳ χρόνῳ . . . ἅπτεσθαι.   I read with Apelt ὡς ἐν πεπερασμένῳ . . . ἅπτεται.   And in l. 30 I accept Apelt's τὰς αὐτὰς ἔχει διαιρέσεις (for which he compares Arist. *Phys.* 235[a] 15) for the MSS. τόσας, or τοσαύτας, ἔχει διαιρέσεις.

[2] [a]26-30. The period and the path of the motion, *quâ* continuous quanta, are divisible *ad infinitum* : but, *quâ* determinate (finite), may both be regarded as containing a finite number of units, i.e. as admitting a finite number of divisions only.   Zeno's argument depends on the fallacy of viewing the period as finite, and neglecting its divisibility *ad infinitum quâ* continuous : whilst the path is viewed (*quâ* continuous) as an actual infinity of points, and its finiteness is neglected.   [Cf. also Aristotle's solution of Zeno's argument, *Phys.* 233[a] 8-34.]

that, because you are unable to solve Zeno's argument, you should make yourselves slaves of your inability, and should commit yourselves to still greater errors, in the endeavour to support your incompetence.[1]

6 (v) As to what they say about 'commensurate lines'—that all lines, because commensurate [2], are measured by one and the same actual unit of measurement—this is sheer sophistry; nor is it in the least in accordance with the mathematical assumption as to commensurability. For the mathematicians do not make the assumption in this form, nor is it of any use to them.

10 Moreover, it is actually [3] inconsistent to postulate both that every line becomes commensurate, and that there is a common measure of all commensurate lines.[4]

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[1] This and the preceding argument are directed against the fourth argument (968ᵃ 18–ᵇ4) of the advocates of indivisible lines.

The writer urges (i) that Zeno's argument involves a fallacy, which the advocates of indivisible lines have failed to detect (969ᵃ 26–30). (ii) That the movement of thought ('psychical process') is not analogous to the movement of a body. The latter is essentially conditioned by the continuity of the path traversed and the continuity of the body moving: for physical movement takes place in a material *substratum*—i.e. a solid material body—and along a path in space. (iii) That if the movement of thought were analogous to the movement of a body, more than this would be required to constitute 'counting'. For to 'count' is not merely to traverse a continuous path, coming into instantaneous contact with the infinite succession of points, into which that path may be mathematically resolved: to 'count' essentially involves *pausing* at the successive steps of the process. (iv) That the argument drawn from 'counting' is an extravagant supposition by which the advocates of 'indivisible lines' are endeavouring to support themselves in an erroneous position—a position really due to their incompetence in failing to detect Zeno's fallacy.

[2] ᵇ7. The MSS. read ὡς ὅτι αἱ πᾶσαι. This presumably means 'e.g. that' or 'viz. that'. But it is very doubtful whether ὡς ὅτι could be used in this way as equivalent to the ordinary οἷον ὅτι. I propose to read ὡς, ὅτι ⟨σύμμετροι⟩, αἱ πᾶσαι . . . .

[3] καὶ ἐναντίον.

[4] ᵇ6–12. This is directed against the fifth argument of the advocates of indivisible lines (cf. above, 968ᵇ 4–14).

It is difficult to be sure of the meaning of 969ᵇ 10–12, owing to the obscurity of the argument which is being attacked. I think the point of the criticism is as follows. The mathematical definition of commensurate lines can always be satisfied, in the sense that, given any line *AB*, you can always find a line 'commensurate' with it: i.e. any line can become 'commensurate' with *some* line. But though *all* lines are 'commensurate' in this sense, they are not all commensurate *with one another*, and have not got one and the same common measure. Yet the advocates of 'indivisible' lines maintain *both* (i) that any line can become 'commen-

Hence their procedure is ridiculous, since, whilst professing 12
that they are going to demonstrate their thesis in accordance
with the opinions of the mathematicians, and by premisses
drawn from the mathematicians' own statements, they lapse
into an argument which is a mere piece of contentious and
sophistical dialectic—and such a feeble piece of sophistry too !
For it *is* feeble in many respects, and totally ⟨unable⟩ to
escape paradox on the one side, and destructive scientific
criticism on the other.[1]

Moreover, it would be absurd for people to be led astray by 16
Zeno's argument, and to be persuaded—because they cannot
refute it—to invent indivisible lines : and yet to pay no atten-
tion to all those theorems concerning lines, in which it is proved
that it is impossible for a movement to be generated such that
in it the moving thing does *not* fall successively on each of
the intervening points before reaching the end-point.   For the 25
theorems in question are far better established, and more
generally admitted, than the arguments of Zeno.[2]

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surate ', *and* (ii) that all commensurate lines have a common measure :
and these two propositions are inconsistent.   For (i) is true only if
' commensurate ' be used in a *relative* sense ; and then (ii) is false.
Whilst (ii) is true only if ' commensurate ' be used in an absolute sense ;
and then (i) is false.

[1] ᵇ12–16.   Bekker reads ὥστε γελοῖον τὸ [τὸ om. Wᵃ] κατὰ [καὶ N] τὰς
ἐκείνων δόξας καὶ ἐξ ὧν αὐτοὶ λέγουσι φάσκοντες δείξειν, εἰς ἐριστικὸν ἅμα καὶ
σοφιστικὸν ἐκκλίνειν [ἐγκλίνειν LPWᵃ ἐγκλίναι N] λόγον, καὶ ταῦθ' οὕτως
ἀσθενῆ.   πολλαχῇ [πολλαχῶς LPWᵃ] γὰρ ἀσθενής ἐστι καὶ πάντα τρόπον
διαφυγεῖν καὶ τὰ παράδοξα καὶ τοὺς ἐλέγχους.
  By reading φάσκοντας in l. 13 very tolerable sense may be made of the
first sentence.   Apelt follows N and reads τὸ καὶ τὰς κτλ ... ἐγκλίναι ...
' ridiculum est et illorum (sc. mathematicorum) placita et ea, quibus ipsi
argumenta sua superstruunt, in sophisticas captiones detorquere.'   But
αὐτοί (cf. 968ᵇ 4, to which this refers) is most naturally taken as ' the
mathematicians ': and in any case Apelt's interpretation is not con-
vincing.
  The last sentence seems to be corrupt.   The general sense of the
passage would be satisfied by πάντα τρόπον ἀδύνατος (or ἀδυνατεῖ) διαφυγεῖν
... : but I hesitate to propose any reading.   The point seems to be that
the advocates of indivisible lines are exposed to a double fire.   They are
using as an argument what to common sense is ridiculous paradox, and
what to professional mathematicians is demonstrably unscientific.

[2] ᵇ16–26.   In the above paraphrase I think I have reproduced the
general drift of this passage.   Zeno showed that if a body is to move from
A to *B*, it must touch all the intermediate points before reaching *B* : i.e.
it must traverse an infinity in a finite time.   And he argued that motion
is impossible.   The advocates of indivisible lines replied : ' Motion is a

26   § 3. It is clear, then, that the being of indivisible lines is neither demonstrated nor rendered plausible—at any rate by the arguments which we have quoted. And this conclusion will grow clearer in the light of the following considerations :—

29   (A) In the first place,[1] our result will be confirmed by reflection on the conclusions proved in mathematics, and on the assumptions[2] there laid down—conclusions and assumptions

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fact, and therefore—since Zeno's argument is sound—the line *AB* must consist of a finite number of indivisible unit-lines.' The writer here rejoins : 'Geometry proves that there can be no motion without the phenomenon to which Zeno called attention. A motion, such as your theory requires—a motion in which the moving body does not traverse successively all the intermediate points—does not, and cannot, occur. And the theorems, in which geometry establishes this, are far more convincing than the arguments of Zeno.'

In other words :—Geometry, assuming motion to be a fact, shows that the moving thing *does* traverse an infinity of intervening points, and shows that there can be no motion in which this does not take place. The advocates of indivisible lines have made no attempt to refute these geometrical proofs. Their postulate of 'indivisible lines', even if it evaded Zeno, collides with these far more solid facts of geometry : for the kind of motion which would occur, if there were indivisible lines, is shown by geometry to be impossible.

The text of this passage is so corrupt that it seems hopeless to make out the details of the argument.

In ll. 19-21 the writer is clearly referring to the movement of a straight line about one of its terminal points, whereby a semicircle (and, ultimately, a circle) is generated. διάστημα is the regular term in Euclid for the distance at which, from a given point as centre, the circumference of a circle is drawn. Cf. e.g. Eucl. *Elem.* I. 22 . . . κέντρῳ μὲν τῷ Z, διαστήματι δὲ τῷ ZΔ κύκλος γεγράφθω ὁ ΔΚΛ . . ., and so constantly. (διάστημα in fact = 'radius'.)

In l. 19 we should read with Apelt διὰ δὲ⟨τὴν⟩ τῆς εὐθείας εἰς τὸ ἡμικύκλιον [so NZ²: the other MSS. read ἡμιόλιον] κίνησιν, . . .

But Apelt (in the *Prolegg.* to his text) proposes other emendations for the rest of the passage, which are not convincing. It is best to recognize that the passage is hopeless, until somebody can discover the exact geometrical theorems to which the writer is referring.

[1] ᵇ28 ff. The writer is going to show that the doctrine of indivisible lines cannot be reconciled with mathematics. It collides with the conclusions established in mathematics, and it collides with the premisses laid down by the mathematicians. He adduces a series of instances of such collision, and sums up at 970ᵃ 17 ἄλλα δ' ἄν τις καὶ ἕτερα τοιαῦτα συνάγοι· πᾶσι γὰρ ὡς εἰπεῖν ἐναντιοῦται τοῖς ἐν τοῖς μαθήμασιν.

πρῶτον μέν (ᵇ29) is answered by πάλιν (970ᵃ 19).

[2] ᵇ30. I have translated τιθεμένων 'assumptions'. It probably includes (a) definitions of the meaning of 'Subjects' and 'Attributes' (=Aristotle's ὁρισμός, where that is used in a restricted sense and contrasted with ὑπόθεσις : cf. e.g *Post. Anal.* 72ᵃ 21-24), and (b) Aristotle's ἴδιαι ἀρχαί, i.e. definitions of the meaning of the 'Subjects' accompanied by the ὑπόθεσις ὅτι ἔστι (cf. e.g. *Post. Anal.* 76ᵃ 32-36).

which we have no right to reject except on more convincing arguments than those adduced by the advocates of indivisible lines.[1]

For (i) neither the definition of ‘line’, nor that of ‘straight line’, will apply to the indivisible line, since the latter is not between any terminal points, and does not possess a middle.[2]

(ii) Secondly, all lines will be commensurate. For all lines **970ᵃ** —both those which are commensurate in length, and those which produce commensurate squares—will be measured by the indivisible lines.

And the indivisible lines are all of them commensurate in length (for they are all equal to one another), and therefore also they all produce commensurate squares. But if so, then the square on any line will always be rational.[3]

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[1] ᵇ30, 31. I read with Apelt (after NZᵃ) ἃ δίκαιον ἢ μένειν ἢ πιστοτέροις λόγοις κινεῖν.

Since obviously the mathematician adduces no arguments in support of his τιθέμενα, I have interpreted πιστοτέροις as above. (It is possible, however, that we should translate ‘more convincing than the mathematical statements’: cf. *de Caelo* 299ᵃ 5 καίτοι δίκαιον ἦν ἢ μὴ κινεῖν ἢ πιστοτέροις αὐτὰ λόγοις κινεῖν τῶν ὑποθέσεων.) The writer lays down the general principle that we are bound to accept the assumptions and conclusions of the mathematician in the sphere of mathematics, unless very convincing arguments are brought against them.

[2] ᵇ31–33. The first instance adduced by the writer to show that the theory of indivisible lines collides with τὰ ἐν τοῖς μαθήμασι τιθέμενα.

We must suppose that it was customary in contemporary mathematics to define *line* as ‘that which is between two points’, and *straight line* as ‘that, the middle point of which is in the way of [blocks] both ends’. For *the first definition*, cf. perhaps Arist. *Phys.* 231ᵇ 9, στιγμῶν δ’ ἀεὶ τὸ μεταξὺ γραμμή. For *the second definition*, cf. perhaps Plato, *Parmen.* 137 E, where that σχῆμα is said to be εὐθύ “οὗ ἂν τὸ μέσον ἀμφοῖν τοῖν ἐσχάτοιν ἐπίπροσθεν ᾖ”.

[3] At 970ᵃ 4 I accept Apelt’s conjecture, ἀεὶ ῥητὸν ἔσται τὸ τετράγωνον for the MSS. διαιρετὸν ἔσται τὸ τετράγωνον.

This second instance (969ᵇ 33–970ᵃ 4), in which the doctrine collides with mathematics, is a case partly of collision with the definitions of certain mathematical properties, partly of collision with certain demonstrated conclusions.

The writer complains that the doctrine of indivisible lines plays havoc (i) with the mathematical definition of ‘commensurate’ lines, and the mathematical distinctions which follow from it ; for since all lines whatever consist of a whole number of these unit-lines, it follows that all lines are commensurate μήκει, and the mathematical distinction between surds and rational roots vanishes (969ᵇ 33–970ᵃ 2) : and (ii) with the mathematical definition of ‘rational’ squares, and the distinction between ‘rational’ and ‘irrational’ squares which follows from it. For the indivisible lines

4   (iii) Again, since, in a rectangle, the line applied at right angles to the longer side determines the breadth of the figure: the rectangle, which is equal in area to the square on the indivisible line (v.g. on the line one foot long), will, if applied to a line double the indivisible line (v.g. to a line two feet long), have a breadth determined by a line shorter than the indivisible line : for its breadth will be less than the breadth of the square on the indivisible line.[1]

are all, *quâ* infinitesimal, equal : hence all commensurate μήκει, and therefore also commensurate δυνάμει (970ᵃ 2–4).

The point of the criticism is that the doctrine annihilates the mathematical conceptions of Commensurate and Incommensurate, Rational and Irrational.

The passage should be compared with Euclid, *Elem.* X, deff. 2, 3 and 4 (see above, note on 968ᵇ14) : and with Plato, *Theaet.* 147 D–148 B. In the *Theaetetus*, Theaetetus and Socrates the Younger are represented as having generalized certain results of the mathematician Theodorus (their master), and having divided all numbers into two series, thus :—

Series 1 : Those numbers which, if regarded as the areas of rectangular figures, are squares with whole numbers as their sides, e.g. 4, 9, 16, 25, &c.

The roots of these square numbers are what we should call 'rational' : or the sides of the squares are lines σύμμετροι μήκει, viz. containing whole numbers of the unit of length (the line one foot long).

Theaetetus and Socrates called the sides containing the squares in this series 'μήκη'.

Series 2 : Those numbers which, if regarded as the areas of rectangular figures with whole numbers as their sides, are oblongs ; or, if regarded as squares, have not whole numbers as their sides. To this series belong e.g. 3, 5, 6, 7, 8, &c. : and the sides containing these squares—e.g. $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, &c.—were called by Theaetetus and his friend 'δυνάμεις', i.e. δυνάμει σύμμετροι. (Cf. *Theaet.* 147 D ἥ τε τρίπους καὶ ἡ πεντέπους δυνάμεις are not μήκει σύμμετροι τῇ ποδιαίᾳ. *Ib.,* 148 B ὡς μήκει μὲν οὐ συμμέτρους ἐκείναις, τοῖς δ᾽ ἐπιπέδοις ἃ δύνανται.)


We should call the 'sides' of this series of squares 'irrational square roots' or 'surds'.

[1] ᵃ4–8. In this passage I adopt Apelt's reading and interpretation throughout: *v.* Apelt, *Aristotelis quae feruntur,* &c., *prolegg.* pp. xiv, xv.

If we suppose the 'indivisible line' to be one foot long (cf. Arist. *Met.* 1052ᵇ 33—ἐν ταῖς γραμμαῖς χρῶνται ὡς ἀτόμῳ τῇ ποδιαίᾳ), then a rectangle, applied to a line two feet long, *must*—if its area is to be equal to the square on the indivisible line—have as its other side a line shorter than the indivisible line : which is absurd.

Let *AB* be the indivisible line, one foot long. Let *BE* be the line, two
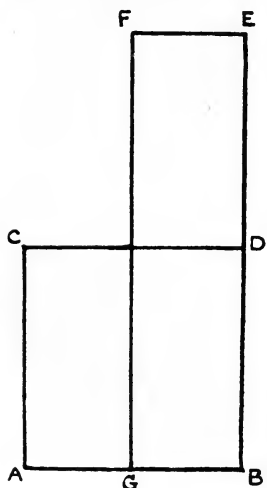
(iv) Again, since any three given straight lines can be com- 8 bined to form a triangle, a triangle can also be formed by combining three given indivisible lines. Such a triangle will be equilateral : but in every equilateral triangle the perpendicular dropped from the apex bisects the base. Hence, in the equilateral triangle whose sides are the indivisible lines, the 'indivisible' base will be bisected by the perpendicular dropped from its apex.[1]

(v) Again, if the square can be constructed of Simples (i.e. 11 with indivisible lines as its sides), then let its diagonal be drawn, and a perpendicular dropped from one angle on to the diagonal. The square on the side (i.e. the original square constructed

feet long. Let *CABD* be the square on *AB*. If to the line *BE* there be applied a rectangular figure *GFEB* equal in area to *CABD*, *FE* or *GB* will be less than *AB*.

Though I accept Apelt's interpretation, there are one or two difficulties to which attention should be called. (1) παραβάλλειν is the technical term constantly used in Euclid (cf. e.g. *Elem.* I. 44, &c.) for 'applying' a rectangle or a parallelogram to a given line : i.e. for constructing such a figure with a given line as one of its sides. But (so far as I know) it is always the figure which 'παραβάλλεται', not the side. Hence παραβαλλομένη here (970ᵃ 5) is suspicious.

(2) Euclid constantly uses the technical expression 'πλάτος ποιεῖ τὴν *AB*' to mean '[a rectangle applied to such-and-such a given line] makes as its other side the line *AB*'. But, whatever may have been the original significance of the phrase, there is no implication in Euclid's usage that the side thus produced is *shorter* than the given line. So far as I have been able to discover, πλάτος ποιεῖν in Euclid (*a*) always has the accusative (e.g. 'τὴν *AB*') expressing the line resulting, and (*b*) does not mean 'determines the breadth', but simply 'makes as its containing side (other than the given line)'. Cf. e.g. Euclid, *Elem.* X. 60, where the line thus produced is the longer of the two containing sides : and so often. But here (970ᵃ 5, ᵃ7) the writer speaks of a line 'making the breadth' (τὸ πλάτος ποιεῖ), and the expression must be distinguished from the technical phrase in Euclid.

(3) In 970 ᵃ6 Apelt reads τῷ ἀπὸ τῆς ἀτόμου καὶ τῆς ποδιαίας. τὸ ἀπὸ τῆς ἀτόμου means 'the square on the indivisible line'(cf. above, note on 968ᵇ 14): and we are to take the καί as illustrative or explanatory. There is no serious difficulty here, though this introduction of the one-foot line is a little sudden. But the words in l. 8 are very difficult. Apelt there reads ἔσται ⟨γὰρ⟩ ἔλαττον τοῦ ἀπὸ τῆς ἀτόμου, and the words ought to mean 'For it'—presumably, 'the breadth'—'will be less than the square on the indivisible line'. As this is nonsense, and as the alternative rendering ('for it', viz. *the rectangle*, 'is less than the square') gives a meaning irrelevant to the argument, we have to translate 'For the breadth of the rectangle will be less than that of the square'. But I cannot say that the Greek justifies this translation.
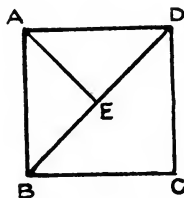
[1] ᵃ8–11. This argument presents no difficulty. Cf. Euclid, *Elem.* I. 10. συνίστασθαι is the regular term in Euclid for 'constructing' a figure.

with Simples as its sides) will be equal to the square on the
perpendicular together with the square on half the diagonal.
Hence the side of the square—i.e. the 'indivisible' line—will
not be the smallest line.[1]

14   Nor will the area, which is the square on the diagonal, be
double the square on the indivisible line. For ⟨suppose it to
be so : then,⟩ if from the diagonal a length equal to the side
of the original square be subtracted, the remaining portion of
the diagonal will be less than the 'simple' line. For if the
remaining portion of the diagonal were ⟨not less than, but⟩
equal to the 'simple' line, the square on the diagonal would
have been four times the original square.[2]

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[1] ᵃ11–14. I adopt Apelt's emendation διαμέτρου in l. 12 for the MSS. διὰ
μέσου, and in l. 11 I read εἰ τὸ τετράγωνον ⟨ἐκ⟩ τῶν ἀμερῶν (sc. συνίσταται).
The Latin translation by Rota has 'si quadratum ex quatuor insecabilius
lineis consistat', and LPWᵃ omit τετράγωνον in a lacuna. Perhaps we should
read εἰ τὸ τετράγωνον ἐκ τεττάρων ἀμερῶν, or εἰ ἐκ τεττάρων ἀμερῶν τετράγωνον.

Another interpretation would be possible, if we retain the MSS. reading
εἰ τὸ τετράγωνον τῶν ἀμερῶν, but alter ἐλαχίστη in l.14 to ἐλάχιστον. ' If
the square belongs to the class of Simples, then ... [as above] ... half
the diagonal. Hence the " simple " square will not be the smallest

square.' The argument would then be directed
against the application of the theory of 'simples' to
squares (cf. above, 968ᵇ14–16). The assumption of a
least 'indivisible' or 'simple' square collides with
Euclid, *Elem.* I. 47. For, let *ABCD* be the
'simple', or 'minimal', square. Draw the dia-
gonal *BD*, and the perpendicular *AE* bisecting
BD at *E*. Then, since *AEB* is a right angle,
$AB^2 = AE^2 + BE^2$, and therefore AE^2 and BE^2 are,
each of them, smaller squares than the supposed
smallest square, *ABCD*.

But the expression in l. 12 (ἡ τοῦ τετραγώνου πλευρά ...), and also the
argument in ll. 14–17, seem decisive in favour of the interpretation which
I have adopted in the text.

[2] ᵃ14–17. In l. 17 I read (with N and Apelt) εἰ γὰρ ἴση, τετραπλάσιον ἂν
ἔγραψεν ἡ διάμετρος. And after διάμετρος I read a full stop.

Geometers have proved (i) that the square on the diagonal = twice the
square within which the diagonal is taken: i.e. that $BD^2 = 2ABCD$:

and (ii) that if any line *xy* = twice any other line
mn, $xy^2 = 4mn^2$.
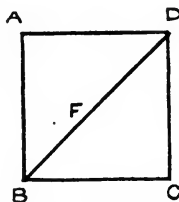
Hence, it follows that *BD* in the square *ABCD*
is less than $2AB$: i.e. that, if from *BD* a portion
$DF = AB$ be subtracted, the remainder *BF* is less
than *AB*. If, therefore, *AB* is an 'indivisible' line,
either BD^2 will not be equal to $2AB^2$ (but = at
least $4AB^2$), *or* *BD* will contain *FD* (= *AB*)
+ *BF* (a line less than the 'indivisible' line) : the
first alternative conflicts with an established geo-
metrical conclusion, and the second alternative is absurd.

And one might collect other similar absurdities to which the doctrine leads; for indeed it conflicts with practically everything in mathematics.[1]

(B) Then again ⟨the following arguments support our criticism of the doctrine⟩ :—[2]

(i) The Simple admits of only one mode of conjunction, but 19 a line admits of two : for one line may be conjoined to another either by contact along the whole length of both lines, or by contact at either of its opposite terminal points.[3]

(ii) Further, the addition of a line will not (on the theory) make the whole line any longer than the original line to which the addition was made : for Simples will not, by being added together, produce an increased total magnitude.[4]

(iii) Further, every continuous quantum admits more divi- 23 sions than one, and therefore no continuous quantum can be formed out of two Simples. And since every line (other than the indivisible line) is admittedly continuous, there can be no indivisible line : ⟨for if there were, a continuous quantum— viz. the line formed by the conjunction of two indivisible lines —would be formed out of two Simples.⟩[5]

In l. 16 ἀφαιρεθέντος γάρ τοῦ ἴσου, we should presumably understand μήκους.

[1] ᵃ17. The MSS. read ἄλλα δ’ ἄν τις καὶ ἕτερα κτλ. Apelt conjectures ἄλογα δ’ ἄν κτλ. There should, of course, be a full stop between διάμετρος and ἄλλα (or ἄλογα).

[2] ᵃ19. This begins a second series of arguments (in support of the writer's rejection of indivisible lines). πάλιν here corresponds to πρῶτον μὲν ... (969ᵇ 29), which introduced the series of arguments just concluded.

[3] ᵃ19–21. What is ‘simple’ or ‘without parts’ can be conjoined with anything else only in one fashion. But a line can be (a) laid alongside of another line, or (b) conjoined with it, end to end. (Cf. de Caelo, 299ᵇ 25).

The words in ᵃ21 κατὰ τὸ πέρας ἐξ ἐναντίας (ἐναντίου LP) are obscure. I take them to mean ‘at either of its contrary terminal points’. The *mode* of σύναψις is the same whether the line xy be conjoined with the line AB at A or at B, and at x or at y.

[4] ᵃ21–23. Apelt conjectures (from Pachymeres) ἔτι γραμμὴ ⟨γραμμῇ⟩ προστεθεῖσα ...

The addition of γραμμῇ makes the Greek easier, but does not seem absolutely necessary.

[5] ᵃ23–26. I adopt Apelt's reading ἔτι ⟨εἰ⟩ ἐκ δυοῖν ἀμεροῖν μηδὲν γίνεται (γίνεσθαι MSS.), and also his punctuation, but not his interpretation.

I have paraphrased freely, so as to bring out the argument as I under-

26   (iv) Further, if every line (other than the indivisible line) can be divided both into equal and into unequal parts—every line, even if it consist of three or any odd number of indivisible lines—it will follow that the ' indivisible ' line is divisible.[1]

stand it. The writer assumes (ἅπασα δὲ γραμμὴ παρὰ τὴν ἄτομον συνεχής) that even the advocates of indivisible lines admit that all *other* lines are continuous : and argues that a line compounded of two indivisible lines would, on their admission, have to be continuous, but could not be so on the principle that every *continuum* admits more than one division.

[1] ᵃ26–28. The MSS. read ἔτι εἰ ἅπασα γραμμὴ παρὰ [περὶ LNP, om. Zᵃ] τῆς ἀτόμου καὶ ἴσα [καὶ εἰς ἴσα L] καὶ ἄνισα διαιρεῖται καὶ μὴ ἐκ τριῶν ἀτόμων καὶ ὅλως περιττῶν ὥστ' ἀδιαίρετος ἡ ἄτομος.

I accept Apelt's reading (which is partly based on Hayduck's conjectures) ἔτι εἰ ἅπασα γραμμὴ παρὰ τὴν ἄτομον καὶ εἰς ἴσα καὶ ἄνισα διαιρεῖται, κἂν ἦ ἐκ τριῶν καὶ ὅλως περιττῶν, ἔσται διαιρετὴ ἡ ἄτομος.
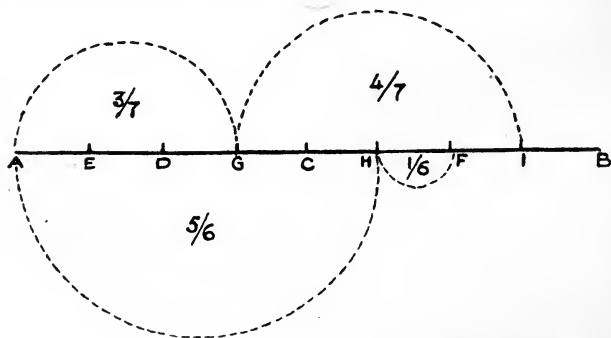
The writer is assuming, in the present series of arguments (970ᵃ 19–33), that the advocates of indivisible lines accept certain common mathematical assumptions as applying to the *composite* (non-indivisible) lines : and shows that their application is inconsistent with the ' indivisibility ' of the unit-lines.

The assumption here stated is ἅπασα γραμμὴ καὶ εἰς ἴσα καὶ ἄνισα διαιρεῖται. This formula is constantly used by Euclid (cf. e.g. *Elem.* II. 5 and 9) to mean *bisection* and simultaneous division into *two* unequal parts. If we so understand it here, the argument is plain : but then l. 33 (ὅταν ἡ ἐκ τῶν ἀρτίων εἰς ἄνισα διαιρῆται) is unintelligible.

It seems best, therefore, to interpret ' into *any number* of equal, and *any number* of unequal parts '. And there is reason for thinking that ' division into unequal parts ', as here contemplated, involved a process of progressive bisection. (Cf. e.g. Alexander's Commentary on Arist. *De Sensu*, 445ᵇ 27 : and G. R. T. Ross, *Aristotle : De Sensu and De Memoria*, pp. 199–200.) If, e.g., the line AB was to be divided into $\frac{1}{4}$ and $\frac{3}{4}$, the method would be to bisect AB at C,

and again to bisect AC at D. AD would then be $\frac{1}{4}$, and DB $\frac{3}{4}$, of AB. It would not be possible by this method to divide AB into parts repre-


sented by fractions whose denominators were other than powers of 2 : but it would be possible to exhibit such fractions *on* the line AB. Thus, e.g.,

And the same will result if every line admits of bisection : 29 for then every line consisting of an odd number of indivisible lines will admit of bisection, and this will involve the division of the 'indivisible' line.[1]

it would not be possible to divide AB into $\frac{3}{7}$ and $\frac{4}{7}$, nor into $\frac{5}{6}$ and $\frac{1}{6}$. But by triply bisecting AB, and eliminating $\frac{1}{8}$th, the remainder AI could be divided into $AG = \frac{3}{7}$ and $GI = \frac{4}{7}$: whilst, by eliminating $\frac{2}{8}$th, the remainder AF could be divided into $AH = \frac{5}{6}$ and $HF = \frac{1}{6}$.

There is no evidence in this passage that the writer knew of the following method for dividing any given line into any number of parts :—Let it be required to divide AB into (e. g.) three equal parts. From B draw BC
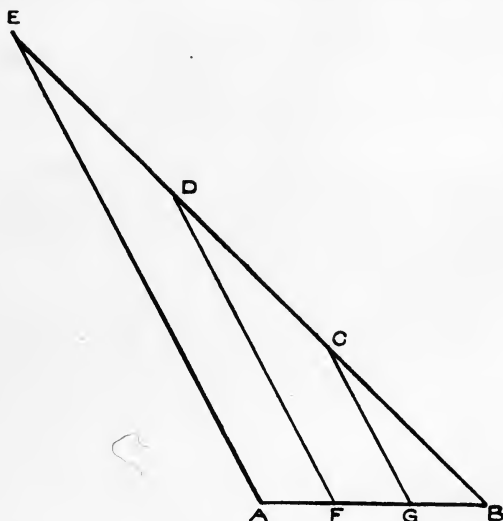
$= AB$, produce BC to D, making $CD = AB$: and produce BD to E, making $DE = AB$. Join EA ; and from D and C draw DF and CG, each parallel to EA, to the points F and G on AB. AF, FG, and GB will then be, each of them, $\frac{1}{3}$rd of AB.

If we assume that the writer was unaware of this latter method, it is obvious (a) that no line consisting of an odd number of unit-lines could be 'divided into unequal parts', for the first bisection would divide the middle unit-line : and (b) that there would be a limit to the 'division into unequal parts' of lines consisting of an even number of unit-lines, since no such line could be progressively bisected ad libitum without dividing the unit-line (cf. 970^a 33).

[1] ^a29, 30. Mathematicians further assume that every line can be bisected. If the advocates of 'indivisible' lines accept this assumption, it will apply to lines compounded of an odd number of unit-lines (πᾶσα γὰρ ἡ ἐκ τῶν περιττῶν, sc. δίχα τέμνεται) : but they cannot be bisected unless the *middle* 'indivisible' line is divided.

And if not *every* line, but only lines consisting of an even number of units admit of bisection : still, even so, the 'indivisible' line will be divided, when the line consisting of an even number of units is divided into unequal parts (by progressive bisection).[1]

33   (C) Again,[2] ⟨the following arguments must be considered against the doctrine⟩ :—

970ᵇ   (i) If a body has been set in motion and takes a certain time to traverse a certain stretch, and half that time to traverse half that stretch, it will traverse less than half the stretch in less than half the time.[3] Hence if[4] the stretch be a length consisting of an odd number of indivisible unit-lines, we shall here again find[5] the bisection of the 'indivisible' lines, since the 5 body will traverse half the stretch in the half time : for the time and the line will be correspondingly divided.[6]

So that none of the composite lines will admit of division both into equal and into unequal parts, nor will they admit of

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[1] ᵃ30–33. In the above interpretation I have omitted altogether the words τὴν δὲ δίχα διαιρουμένην καὶ ὅσα δυνατὸν τέμνειν. These words as they stand will not translate. If we read καὶ εἰς ἄνισα in place of καὶ ὅσα, the meaning is plain enough : but the words are then not required for the argument.

Hayduck, and after him Apelt, conjectures καὶ ὁσαοῦν 'and if it is possible to divide (i. e. to bisect) the line which is being bisected (viz. the line with an even number of units) as many times as you please'. But, if my interpretation of διαίρεσις εἰς ἄνισα is right, these words are not required. Whilst, if my interpretation is wrong, I do not see how a valid argument is to be extracted from the passage. Apelt (cf. his *Prolegg.* p. xviii, note, p. xix : and his German translation of the passage) interprets ἄνισα as equivalent to περιττά, for which I can discover no justification.

[2] ᵃ33. πάλιν εἰ κτλ. This πάλιν answers to πάλιν τοῦ μὲν ἀμεροῦς (970ᵃ 19), and marks the beginning of a new group of arguments.

[3] The protasis extends to κινηθήσεται, and the apodosis is καὶ ἐν τῷ ἐλάττονι . . . ἡμίσειαν. We should therefore place a comma after κινηθήσεται (970ᵇ 2).

[4] ᵇ3. I adopt Apelt's conjecture εἰ μὲν ⟨ἐκ⟩ περιττῶν.

[5] ᵇ3. The MSS. read ἀναιρεθήσεται (Zᵃ fort. ἀνερεθήσεται). Apelt conjectures αὖ εὑρεθήσεται, but the position of the αὖ is impossible. I read ἀνευρεθήσεται ('redibit', Rota).

[6] ᵇ5, 6. Since the time is bisected, the stretch—i. e. the line, supposed in this case to consist of an odd number of units—will be bisected too. After these words there is, I think, a lacuna. For nothing is said as to the case in which the stretch consists of an even number of units :—i.e. there is no clause to answer to εἰ μὲν ἐκ περιττῶν in 970ᵇ 3. And no use is made of the thesis established in 970ᵇ 2 (καὶ ἐν τῷ ἐλάττονι . . . τὴν ἡμίσειαν), which was probably intended to be applied in proving the divisibility of the unit-line, even when the stretch consisted of an even number of units.

division corresponding to the division of the times, if there are to be ' indivisible ' lines.[1] And yet (as we said) the truth is, that the same argument, which leads to the view that lines consist of Simples, leads by logical necessity to the view that all these things (composite times, e.g., as well as composite lines) consist of Simples.[2]

(ii) Further, every line which is not infinite has two terminal 10 points : for line is defined by these. Now, the ' indivisible ' line is not infinite, and will therefore have a terminal point. Hence it is divisible : for the terminal point and that which it terminates are different from one another. Otherwise there will be a third kind of line, which is neither finite nor infinite.[3]

(iii) Further, there will not be a point contained in every 14 line. For there will be no point contained in the indivisible line ; since, if it contains one point only, a line will be a point, whilst if it contains more than one point it will be divisible. And if[4] there is no point in the indivisible line, neither will there be a point in any line at all : for all the other lines are made up out of the indivisible lines.[5]   18

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[1] ^{b}7, 8. I read with Hayduck οὐδ' ὁμοίως τοῖς χρόνοις τμηθήσονται, εἰ [MSS. οὐκ] ἔσονται . . . The whole sentence is intelligible only if we assume that something has dropped out between τμηθήσεται and ὥστε in l. 6 : see the preceding note.

[2] ^{b}8. τὰ δὲ τοῦ αὐτοῦ λόγου ἐστί, καθάπερ ἐλέχθη, τὸ πάντα ταῦτα ποιεῖν ἐξ ἀμερῶν.

The reference is to 969^a 29, 30. For τὰ δέ we should presumably read τὸ δέ. By πάντα ταῦτα we must understand *primarily* μήκη and χρόνοι : but no doubt the statement is intended to apply to *all* composite quanta.

[3] ^{b}10–14. In ^{b}12 I read (with Bekker) ἄλλο for the MSS. ἄλλου. Every line, unless it be infinite, has two ends or limits, viz. its terminal points. The indivisible line, therefore, since it is not infinite, has two limits. But, if it has even *one* limit, it is divisible, viz. into (*a*) the limit, and (*b*) the limited. The only escape from this dilemma ('*either* infinite *or* limited and so divisible') would be to say that the ' indivisible lines ' constitute a third class of line, neither finite nor infinite.

[4] ^{b}17. εἰ μὲν οὖν . . . What is the exact force of ' μὲν οὖν ' here? Does it mean ' And, what is more, if'? Or ' And if it be conceded that'?

[5] ^{b}14–18. In ll. 15–16 I read (with Apelt) εἰ μὲν γὰρ μία μόνη ἐνυπάρξει, γραμμὴ ἔσται στιγμή for the MSS. εἰ μὲν γὰρ μία [μάλιστα LPZ^a] μόνη ὑπάρξει γραμμή, εἶτα στιγμή.

The writer sets out to show that the geometrical principle, that ' in every line there is contained a point ', will not hold of the ' indivisible ' line. For if it contains but one point, it will be that point, i. e. a line will be a point : whilst if it contains more than one, it will be divisible. He then shows that it follows that this geometrical principle does not hold of *any* line, since all lines are (on the theory) either indivisible lines or com-

Moreover, if there are points in the indivisible line, there will either be nothing between the points, or a line. But if there is a line between them, and if all lines contain more points than one, the unit-line will not be indivisible.[1]

21 (iv) Again, it will not be possible to construct a square on every line. For a square will always possess length and breadth, and will therefore be divisible, since each of its dimensions— its length and its breadth—is a determinate something. But if the square is divisible, then so will be the line on which it is constructed.[2]

23 (v) Again, the limit of the line will be a line and not a point.[3] For it is the ultimate thing which is a limit, and it is the 'indivisible line' which is ultimate.[4] For if the ultimate thing be 'point', then the limit to the indivisible line will be a point, and one line will be longer than another by a point.[5] But if it be urged that the limiting point is contained *within* the

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posites of these. For the geometrical principle cf. Arist. *Post. Anal.* 73ᵃ 31 καὶ εἰ ἐν πάσῃ γραμμῇ στιγμή . . .

[1] ᵇ18–20. I interpret this as a further argument to prove that there cannot be two (or more) points in the indivisible line. For suppose there are two points in it. Then *either* there is nothing between them, and then they collapse into an indistinguishable unity: *or* there is a line separating them. But then *this* line will itself contain two or more points, between which there must be another line, and so on *in infinitum*: hence the original unit-line will not be 'indivisible' if it contains two (or more) points.

[2] ᵇ21–23. This argument is very obscure, and perhaps the text is wrong. It is a principle of geometry that a square can be constructed on any given line : but it does not follow, because the length (AB) of the square $ABCD$ is distinguishable from its breadth (AC), and because therefore the square is divisible into length and breadth, that AB or AC are themselves divisible *quâ* lines.

The Greek ἐπεὶ τὸ μέν, τὸ δέ τι seems suspicious, but I have no remedy to propose. Cf., however, the argument at 970ᵇ 30 ff. A square, if divided, must be divided 'at a line' : i.e. its division must involve the division of its breadth or length. But this is impossible if its sides (and therefore all lines within it which are parallel to them) are 'indivisible' lines.

[3] In ᵇ24 I read with Apelt (after Hayduck) γραμμὴ ἔσται, ἀλλ' οὐ στιγμή for the MSS. στιγμὴ ἔσται [ἐστιν N], ἀλλ' οὐ γραμμή. N's ἐστιν is a transparent, but futile, attempt to make sense of the traditional reading.

[4] In ᵇ25 I accept Bussemaker's conjecture τὸ ἔσχατον, ⟨ἔσχατον⟩ δὲ ἡ ἄτομος.

[5] In ᵇ25 I retain the MSS. reading εἰ γὰρ στιγμὴ [sc. τὸ ἔσχατον], τὸ πέρας τῇ ἀτόμῳ ἔσται στιγμή. Apelt's conjecture, εἰ γὰρ στιγμὴ τὸ πέρας, ⟨πέρας⟩ τῇ ἀτόμῳ ἔσται στιγμή, though it would be convenient, is not necessary.

indivisible line, on the ground that two lines united so as to form a continuous line have one and the same limit at their juncture, then the simple line (i.e. the line without parts) will after all have a limit belonging to it.[1]

28

And, indeed, how will a point differ at all from a line on their theory? For the indivisible line will possess nothing characteristic to distinguish it from the point, except the name.[2]

(vi) Again, if there be indivisible lines, there must, by parity of reasoning, be indivisible planes and solids too.[3] For the being of an indivisible unit in one dimension will carry with it the being of indivisibles in the remaining dimensions too,[4] since it is at a plane that a solid is divided, and at a line that a plane is divided. But there is no indivisible solid: for a solid contains depth and breadth. Hence neither can there be an **971^a** indivisible line.[5] For a solid is divisible at a plane, and a plane is divisible at a line.[6]

30

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[1] ^{b}23–28. τὸ ἔσχατον is the ultimate (or most elementary) thing in the spatial sphere: the not-further-reducible element of extended quanta. On the hypothesis of indivisible lines (the writer urges) this ultimate element of extension is the unit-line, and not the point. If it were the point, then *either* (*a*) the point limits the indivisible line *ab extra*, in which case the addition of a point would increase the length of a line : *or* (*b*) the point, which limits the indivisible line, is *internal* to it : but then the internal limiting point will be a distinguishable part of it, i.e. of that which is *ex hypothesi* without parts (cf. 970^b 12, 13).

In ll. 27, 28 the words διὰ τὸ ταὐτὸ πέρας τῶν συνεχουσῶν γραμμῶν (sc. εἶναι) indicate the grounds on which (*b*) might be maintained. If the line CD be joined to the line AB, so as to make a continuous line AD, B and C become one and the same point, the end of AB and the beginning of CD (cf. Arist. *Phys.* 272^a 10–13).

A————B————D (with B above, C below)

[2] ^{b}29, 30. ὅλως τε [read δὲ with N] τί διοίσει στιγμὴ γραμμῆς ; The writer has just shown that the theory leads to the difficulty that a line must be terminated by a line and not by a point. From this special difficulty he now passes to the general difficulty that, on the theory, there can be no difference between 'point' and 'line', except in name.

[3] ^{b}31. The MSS. read ἔτι [ἔτι εἰ N] ὁμοίως μένει ἐπίπεδον καὶ σῶμά ἐστιν ἄτομον. For μένει Hayduck proposed μήκει, and Apelt μὲν καί. I accept Apelt's conjecture, and agree with Hayduck in reading ἔσται for ἐστιν. In ^{b}33 the MSS. read σῶμα οὐκ ἔσται [ἐστὶν NZ^a] ἀδιαίρετον ... : but we must follow the *editio princeps* and insert δὲ after σῶμα. This δέ will then correspond to the μέν in ^{b}31. I agree with Hayduck and Apelt in reading NZ^a's ἐστὶν in ^{b}33 in place of ἔσται.

[4] ^{b}31, 32. Literally, 'For if one is indivisible, all the others will follow suit.'

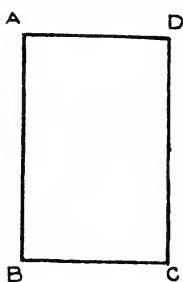[5] ^{a}1. I read with Apelt οὐδ' ⟨ἄρ'⟩ ἂν γραμμὴ εἴη ...

[6] ^{b}30–971^a 3. If there are simple lines, there must be simple planes—viz.

3 But since the arguments by which they endeavour to convince us are weak and false, and since the opinions ⟨which they are trying to establish⟩ conflict with all the most convincing arguments, it is clear that there can be no indivisible line.[1]

§ 4. And it is further clear from the above considerations that a line can no more be composed of points than of indivisible lines. For the same arguments, or most of them, will apply equally against both views.

7 For (i) it will necessarily follow that the point is divided, when the line composed of an odd number of points is divided into equal parts, or when the line composed of an even number of points is divided into unequal parts.[2]

the planes bounded by those lines—and if there are simple planes there must be simple solids, viz. the solids contained by those planes. For to divide a solid is to divide it at a plane, and thus to divide all the planes at right angles to the plane of division. And to divide a plane (cf. above, 970ᵇ 21–23) is to divide it at a line, and thus to divide all the lines at right angles to the line of division. Hence if every solid, however minute, is divisible, every plane must be divisible too : and if every plane, however small, is divisible, every line must be divisible too.

This appears to be the argument : but the reason given (971ᵃ 1) for the divisibility of every solid is obscure, in the same way as the reason given for the divisibility of every square (970ᵇ 23) was not convincing. And could not the advocates of 'indivisible lines' have insisted that a plane figure, though divisible, might yet have as *one* of its containing sides an 'indivisible line'? The oblong *ABCD*, e.g., might be divisible along its length *AB*, and yet indivisible in respect to its breadth *AD* : i.e. *AD* might be an 'indivisible line'.

[1] ᵃ3–5. This sums up the case against the indivisible lines. We have seen in § 2 that the arguments advanced in support of the theory are weak and false : and we have seen in § 3 that the tenets of the theory collide with the principles and conclusions of mathematics.

The text in these lines is not very satisfactory. We should expect a somewhat stronger particle than δέ in ᵃ3 to introduce a summing-up of this kind : but it is difficult to make a convincing emendation. The τε (οἵ τε λόγοι) is apparently answered by δέ in l. 4 (ἐναντίαι δὲ δόξαι), which is omitted by all the MSS. except N. Perhaps the grammatical structure is οἱ λόγοι . . . ἀσθενεῖς τε καὶ ψευδεῖς εἰσι ? See Bonitz, Index, 749ᵇ 44 ff.

All the MSS. in l. 4 read πᾶσαι except P, which has πᾶσι. Neither reading is entirely satisfactory. There seems no point in πᾶσαι, and πᾶσι is not strictly true—or at least has not been shown to be true.

τοῖς ἰσχύουσι [sc. λόγοις] πρὸς πίστιν—'the arguments strong to produce conviction' are presumably the mathematical arguments : cf. e.g. 969ᵇ 30.

[2] ᵃ7–9. I adopt Hayduck's conjecture ἢ ⟨ἡ⟩ ἐκ περιττῶν and ἢ ⟨ἡ⟩ ἐξ ἀρτίων . . .

And (ii) it will follow that the part of a line is not a line, nor the part of a plane a plane.[1]

Further (iii) it will follow that one line is longer than another 10 by a point[2] : for it is by its constituent elements that one line will exceed another. But that it is impossible for one line to be longer than another by a point, is clear both from what is proved in mathematics and from the following argument. For, if it were possible, the absurd consequence would result that the moving body would take a time to traverse the point.[3] For, as it traverses the equal line in an equal time, it will traverse the longer line in a greater time : and that by which the greater time exceeds the equal time is itself a time.

Perhaps, however, we are to suppose that just as a line con- 16 sists of points, so also time consists of ' nows ', and both theses belong to the same way of thinking. ⟨Let us then examine the doctrine that a line, or generally *continua*, like times and lengths, consist of discrete elements.⟩[4]

In l. 9 τὰ ἄνισα is strange : Zᵃ omits τά.

The reference is to the obscure argument at 970ᵃ 26–33.

[1] ᵃ9, 10. If a line is made up of points, a plane on the same principle will be made up of lines : and the ' parts ' of a line will be its ' points ', and of a plane its ' lines '.

[2] ᵃ10, 11. The MSS. read καὶ γραμμὴ δὲ γραμμῆς στιγμῇ [στιγμὴ Wᵃ, στιγμῆς N] εἶναι μείζων·

I read, with Hayduck, καὶ γραμμὴν δὲ γραμμῆς στιγμῇ εἶναι μείζω·

[3] ᵃ13. τὴν στιγμήν, i.e. *the* point, by which the longer line exceeds the shorter. I accept Hayduck's διέναι for the MSS. δὴ εἶναι.

[4] The writer is led off, by a possible rejoinder, to consider the view that time consists of ' nows '. But in the series of arguments which follows, the first argument alone directly mentions ' time ' and ' nows ': and though some of the subsequent arguments would apply to ' time ' as well as to the line, many of them apply specially and only to lines. Hence I interpret 971ᵇ 3 and 4 as a *corollary*, and not as a summary: and I regard *the whole of* § 4 (971ᵃ 6–972ᵃ 13) as a connected series of arguments to show that a line cannot consist of points. The order of the writer's thought is, I think, as follows :—

(1) 971ᵃ 6–16. Statement of the arguments which are fatal *both* to the doctrine that a line consists of indivisible lines, and to the doctrine that it consists of points : and statement of a new difficulty against the latter doctrine. This difficulty involves the conception of Time, and might be met by the rejoinder that Time, like Length, though continuous, consists of discretes. (2) 971ᵃ 17–972ᵃ 13. A group of arguments to show that a line cannot consist of points, the view that Time consists of Nows being incidentally refuted. This group of arguments is based on a disjunction, thus :—The points cannot be united to form the line *either* (a) by συνέχεια (971ᵃ 17–20), *or* (b) by σύνθεσις (971ᵃ 20–26), *or* (c) by ἁφή (971ᵃ 26–ᵇ26), *or* (d) by τὸ ἐφεξῆς (971ᵇ 26–972ᵃ 6).

17   (a) Since, then, the Now is a beginning and end of a [1] time, and the Point a beginning and end of a line; and since the beginning of anything is not 'continuous' with its end, but they have an interval between them; it follows that neither Nows nor Points can be continuous with one another.[2]

20   (b) Again, a line [3] is a magnitude: but the 'composition' of points constitutes no magnitude, because several points put together occupy no more space than one. For when one line is superimposed on another and coincides [4] with it, the breadth is in no wise increased. And since points too are contained in the line thus superimposed, it follows that neither would points, by being superimposed on points, occupy more space. Hence points would not constitute a magnitude by composition.[5]

Of these four alternatives σύνθεσις is used by Aristotle as the general term to express any kind of combination of a manifold: cf. e.g. *Top.* Z 13, 150ᵇ 22, Z 14, 151ᵃ 20–32. Here, however, as we shall see, the writer appears to use it to express one special kind of combination. The remaining alternatives are treated by Aristotle as exhausting the ways in which points might be supposed to cohere to form a line: cf. Arist. *Phys.* 231ᵃ 18 ff. Aristotle's definitions (*Phys.* l. c.), which the writer here assumes, are ' συνεχῆ μὲν ὧν τὰ ἔσχατα ἕν, ἁπτόμενα δ' ὧν ἅμα, ἐφεξῆς δ' ὧν μηδὲν μεταξὺ συγγενές '.

 [1] ᵃ18. τοῦ χρόνου, i.e. any given period of time.

  [2] ᵃ17–20. Two things are called 'continuous' when the end of one is identical with the beginning of the other. But the Nows and the Points are themselves Ends and Beginnings, or Extremes (ἔσχατα), and cannot therefore be ' continuous ' with one another.

  [3] ᵃ21. ἡ μὲν γραμμή ' the line ', i.e. any and every line: cf. 971ᵃ 18, τοῦ χρόνου.

  [4] ᵃ23. For this use of ἐφαρμόζειν cf. e.g. Euclid, *Elem.* I. 4, "ἐφαρμόσει καὶ τὸ Β σημεῖον ἐπὶ τὸ Ε . . ."

  [5] ᵃ20–26. In this argument the writer seems to be excluding a view that point is applied to point so as to 'compound' a line. Line is length without breadth: and if line be applied to line, the two coincide, fall on one another, and do not produce a surface, i.e. do not 'increase the breadth' of the first line. So point is position without magnitude, and no application (composition or addition) of point to point can produce magnitude—i.e. length. If the line *AB* be applied to the line *CD*, the points in *AB* will coincide with the points in *CD*: and as the line *CD* is no 'broader' than it was before, neither will any point *x* in *CD* become a length by ' composition ' with

the corresponding point *y* in *AB*. There is some difficulty in the text. In 971ᵃ 22 the MSS. read διὰ τὸ μηδ' ἐπὶ πλείω τόπον ἔχειν. Should we perhaps read διὰ τὸ μηδ' ἔτι πλείω τόπον κατέχειν? In l. 24 I retain the MSS. reading ἐν δὲ τῇ γραμμῇ . . . (Apelt's emendation εἰ δὲ τῇ γραμμῇ . . . does not suit the movement of the argument.) But I read
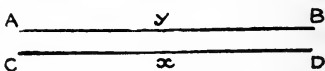
(c) Again, whenever one thing is 'contiguous' with another, 26 the contact is either whole-with-whole, or part-with-part, or whole-with-part. But the point is without parts. Hence the contact of point with point must be a contact whole-with-whole.[1]

But if one thing is in contact with another whole-with-whole, the two things must be one. For if either of them is anything in any respect in which the other is not, they would not be in contact whole-with-whole.[2]

But if the Simples ⟨when in contact⟩ are ⟨not ' one ', but⟩ 30 ' coincident ', then a plurality occupies the same place which was formerly occupied by one : for if two things are coincident and neither admits of being extended beyond the coincidence, just so far the place occupied by both is the same. And since **971ᵇ** the Simple has no dimension, it follows that a continuous magnitude cannot be composed of Simples. Hence neither can a line consist of Points nor a time of Nows.[3]

in l. 25 οὐδ' ἂν ⟨ἄρ'⟩ αἱ στιγμαὶ . . ., and alter the punctuation, so that the whole passage runs as follows :—

. . . μεῖζον τὸ πλάτος· ἐν δὲ τῇ γραμμῇ καὶ στιγμαὶ ἐνυπάρχουσιν· οὐδ' ἂν ⟨ἄρ'⟩ αἱ στιγμαὶ πλείω κατέχοιεν τόπον, ὥστε οὐκ ἂν ποιοῖεν μέγεθος.

[1] In ᵃ27, 28 I read with Apelt (after Hayduck) ἡ δὲ στιγμὴ ἀμερής, ὅλως ⟨ἂν⟩ ἅπτοιτο.

The principle that all contact must be whole-with-whole, or part-with-part, or whole-with-part, is enunciated by Aristotle (*Phys.* 231ᵇ 2), and applied similarly to ἀδιαίρετα and specially to points.

[2] ᵃ29. The MSS. read εἰ γάρ τι [τις NZᵃ] ἐστὶν ἢ θάτερον μή ἐστιν . . .: I read ἢ θάτερον (cf. the Latin transl. ' si quid remanet quod alteri non coniungatur ').

Apelt conjectures εἰ γὰρ δὶς (or δύ') ἐστὶν . . . ' si totum bis est vel non simul alterum complectitur . . .'

[3] ᵃ26–ᵇ4. The outline of the argument is as follows :—The contact of Points, *quâ* Simples, must be whole-with-whole. Now two things are ' contiguous ' when their extremities are ἅμα, ' coincident ' or ' together '. But since Simples have no parts—no extremities in distinction from the rest of themselves—the contact of Simples must mean absolute unity. If this be denied, and it be maintained that the ' contiguous ' Simples are ' coincident ', but remain ' two ': it will follow that two or more Simples can be ' coincident ' without taking up more place than one Simple, and therefore (since *one* Simple has no dimension, i.e. no inner extension) no continuous magnitude can be composed of Simples. And a corollary of this is, that a line cannot consist of points, nor a time of ' nows '.

In 971ᵇ 1 I read, with LPWᵃZᵃ, ἐπέκτασιν, κατὰ ταῦτα ὁ αὐτὸς κτλ. Apelt's conjecture (ἐπέκτασιν καθ' ἑαυτά, ὁ αὐτὸς . . .) is tempting, but unnecessary.

In 971ᵇ 2 διάστασις=dimension, cf. Bonitz, Index, 189ᵃ 30 ff.

4   (d) Further, if the line consists of points, point will be in
contact with point.   If, then, from K there be drawn the lines
AB and CD, the point B in the line $A(B)K$ and the point C in
the line $K(C)D$ will both be in contact with K.[1]   So that the
points B and C will also be in contact with one another : for
the Simple, when in contact with the Simple, is in contact
whole-with-whole.   So that the points will occupy the same
place as K, and, *quâ* in contact with K, will be in the same
place with one another.   But if they are in the same place
10 with one another, they must also be in contact with one another :
for things which are in the same 'continent' place must be in
contact.[2]   But, if this is so, one straight line will touch another

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[1]   [b]4-6. The writer assumes *for the present* that, if a line is made

up of points, the points within the line are in
contact with one another.   Having laid down
this assumption, he then proceeds (ἐὰν οὖν— :
οὖν is omitted by LP, but is required) to
suppose that from the point K two lines,
each consisting of points, are drawn.   He
calls these lines 'AB' and 'CD'; but it is
clear, from what follows, that the points B
and C are the terminal points of the lines
contiguous to K, i.e. that A and D are
the end-points furthest removed from K.

[2]   [b]7-11. This is directed to prove that, since B and C are in contact
with K, they are also in contact with one another.   The text is corrupt,
and I have ventured to read and punctuate as follows :—

ὥστε καὶ ἀλλήλοιν [so Apelt for the MSS. ἄλλων, ἄλλῳ, ἄλλῳ τινί.
Hayduck conjectured ἀλλήλων].   τὸ γὰρ ἀμερὲς τοῦ ἀμεροῦς ὅλον ὅλου
ἐφάπτεται· ὥστε τὸν αὐτὸν ἐφέξει τόπον τῷ Κ, καὶ τοῦ Κ ἀπτόμεναι αἱ στιγμαὶ
ἐν τῷ αὐτῷ τόπῳ ἀλλήλαις. εἰ δ' ἐν τῷ αὐτῷ, καὶ ἅπτονται· τὰ γὰρ ἐν τῷ αὐτῷ
τόπῳ ὄντα πρώτῳ [so Hayduck for the MSS. πρῶτα or πρῶτον] ἅπτεσθαι
ἀναγκαῖον.

For the meaning of πρώτῳ, cf. e.g. *Phys.* 209[a] 32 ff., καὶ τόπος ὁ μὲν
κοινός, ἐν ᾧ ἅπαντα τὰ σώματά ἐστιν, ὁ δ' ἴδιος ἐν ᾧ πρώτῳ. . . .   The
'proper' or 'primary' place of a thing is further explained as that which
contains precisely the thing and nothing more, i.e. the continent boundary
of the thing.   Cf. also *Phys.* 226[b] 21-23.

The argument moves thus : 'B and C are in contact with K.   But
B and C are points, i.e. Simples.   And contact of Simples is contact
whole-with-whole, i.e. complete coincidence.   Hence the "continent
place" of B is identical with that of K, and the "continent place"
of K is identical with that of C.   And therefore the "continent place" of
B is identical with that of C.   But this means that B is in contact with C.'

In 971[b] 8 the MSS. read . . . ἐφέξει τόπον τοῦ Κ, καὶ ἀπτόμεναι στιγμαὶ
. . .   Apelt conjectures ἐφέξει τόπον ⟨τῷ Κ· ἔσονται οὖν καὶ αἱ⟩ τοῦ Κ
ἀπτόμεναι στιγμαὶ κτλ.   This involves more change than the reading which
I propose : and, after all, it is not satisfactory.   For the writer shows
that B and C, *quâ points in contact with a third point*, K—i.e. *quâ*

straight line in two points.   For the point (B) in the line AK touches both the point KC and another ⟨viz. the point contiguous to C in the line $K(C)D$⟩.   Hence the line AK touches the line CD in more points than one.[1]

And the same argument would apply not only in the case 14 supposed, where two lines were in contact with one another at the point K, but also if there had been any number of lines touching one another at K.[2]

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in contact with K whole-with-whole—must have one and the same 'continent place' as K, and therefore as one another: and therefore must be in contact with one another.   The nerve of the argument is contained in the words 'and *the points, because* in contact with K': but Apelt's reading could only be translated 'Therefore the points *which are* in contact with K will also be in the same place as one another'. (Apelt's note on l. 9 εἰ δ' ἐν τῷ αὐτῷ . . . 'scribendum potius videtur γὰρ', shows 'that he has failed to follow the writer's argument.)

[1] ^b11–14.   The writer, having proved that the terminal points B and C are in contact at K, shows that the two straight lines BA and CD will be in contact at more than one point—v. g. at x, since C is in contact with x and B with C.


At l. 11 I adopt Hayduck's εἰ δ' οὕτως for the MSS. εἰθ' οὕτως, and I read a full stop before these words.

At ^b12, 13 I read καὶ τῆς ΚΓ (for the MSS. καὶ τῇ ΚΓ) καὶ ἑτέρας . . . Apelt follows Hayduck in reading καὶ ⟨τῆς ἐν⟩ τῇ ΚΓ.   But 'ΚΓ' is the στιγμή ΚΓ, *not* the γραμμή.   If the writer had meant *the line*, he would have written ΚΔ or ΓΔ as in l. 6 or in l. 13 (τῆς ΓΔ).

Finally, in l. 13 I read (with Hayduck and Apelt) ὥστε ἡ ΑΚ in place of the MSS. ὥστε εἰ ἐκ or ἡ ἐκ.

[2] ^b14, 15.   The MSS. read καὶ εἰ μὴ δι' [δὲ N] ἀλλήλων, ἀλλ' ὁπωσοῦν ἥψατο γραμμῆς [ὁποσοῦν ἥψατο γραμμὴ Z^a].   I have adopted Apelt's conjecture καὶ εἰ μὴ δι' ἀλλήλων, ἀλλ' ὁποσαιοῦν ἥψαντο γραμμαί. If this is right, we must suppose a number of lines, e.g. AB, CD, EF, GH, IJ, all drawn from K.   The points A, C, E, G, I, quâ all in contact with K, are all in contact with one another: and also severally in contact with the points x, y, z, p, q.   Hence the lines AB, CD, EF, GH, IJ will be in contact with one another at more points than one.
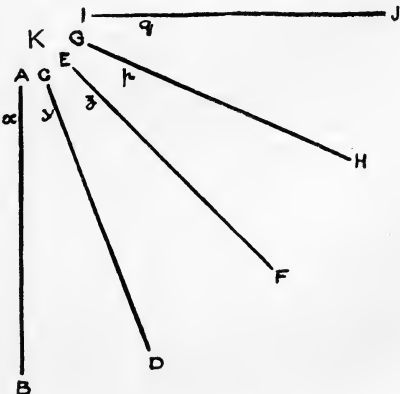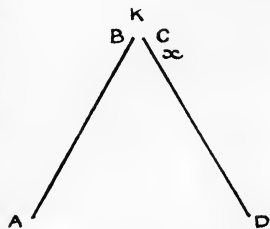
15   (e) Further, if a line consist of points in contact with one another, the circumference of a circle will touch the tangent at more points than one.   For both the point on the circumference and the point in the tangent touch the point of junction and also touch one another.[1]   But since this is not possible, neither is it possible for point to touch point.   And if point cannot touch point, neither can the line consist of points : for if it did, they would necessarily be in contact.[2]

20   (f) Moreover, how—on the supposition that the line consists of points—will there any longer be straight *and* curved lines ? For the conjunction of the points in the straight line will not differ in any way from their conjunction in the curved line. For the contact of Simple with Simple is contact whole-with-whole, and Simples admit no other mode of contact.   Since, then, the straight and curved lines are different, but the conjunction of points is invariably the same, clearly a line will not be curved or straight because of the conjunction : hence neither will a line consist of points.[3]

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[1] ^b15-18. Let the circumference of the circle DEA, and let the tangent CB, both consist of points. The point of juncture, x, will be in contact with the point B of the tangent $CB(x)$, and also with the point A of the circumference $DEA(x)$: hence the point A will also be in contact with the point B. And the tangent $CB(x)$ will touch the circumference $DEA(x)$, at A, at x, and at B.
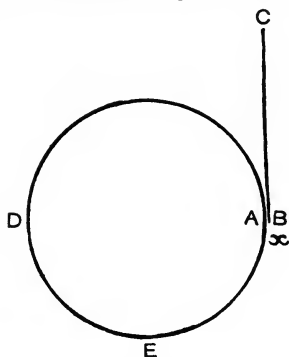
[2] ^b18-20. In l. 20 I read, with Hayduck, οὐδ' εἶναι τὴν γραμμὴν στιγμῶν [MSS. στιγμήν.   Perhaps we ought to read ἐκ στιγμῶν]· οὕτω [MSS. and Apelt οὐδέ] γὰρ ἅπτεσθαι ἀναγκαῖον.

Apelt defends οὐδέ ' si linea ex punctis constaret, necessario a contactu excluderetur (quod tamen fieri nequit) '.   And, in his German translation, he interprets ' Denn sie (die Linie) wäre dann notwendig von der Berührung ausgeschlossen '. But the Greek cannot mean this : nor, if it could, would there be any valid argument in the words.

[3] ^b20-26. In l. 24 I read (with Apelt and Hayduck) ἄλλως ἅπτεσθαι for the MSS. ὅλως [ὅπως W^a] ἅπτεσθαι.

ll. 24-26 are difficult.   I take the writer to mean : ' The theory might attempt to distinguish Straight from Curved, on the ground that point is attached to point differently in these different types of line.   But points are Simples, and therefore point can be attached to point in one way only. Hence we cannot derive the different characters of the straight and curved

(g) Further, the points (of which the line consists) must 26 either touch or not touch one another. Now if 'the next' in a series must touch the preceding term, the same arguments, which were advanced above, will apply : but if there can be 'a next' without its being in contact ⟨with its predecessor or successor⟩, yet by 'the continuous' we mean nothing but a composite whose constituents are in contact. So that the points forming the line must be in contact, in so far as the line must be continuous, even though we suppose the points to be a 'series'.[1]

(h) † ἔτι εἰ ἄτοπον στιγμὴ ἐπὶ στιγμῆς [ἐπιστήμη Zᵃ], ἵν' ᾖ 972ᵃ [ἢ PZᵃ] γραμμὴ καὶ ἐπὶ στιγμῇ, [γραμμὴ καὶ ἐπιστήμης NWᵃ, ἐπιστήμη καὶ γραμμή Zᵃ], ἐπεὶ ἡ γραμμὴ ἐπίπεδον, ἀδύνατον τὰ εἰρημένα εἶναι.[2] † For if the points form a series without

lines from a difference in the mode of contact of their points. And so the theory that lines consist of points in contact breaks down : for it cannot account for the difference between straight and curved.'

In ᵇ25 one may suspect some corruption in the text. The MSS. read οὐκ ἔσται δὴ γραμμὴ ἐκ τῆς συνάψεως. The sense required is given in Rota's translation—'non fiet ex punctorum contactu linea circularis et recta.'

[1] ᵇ26–31. The writer has shown that the points, of which the line is supposed to consist, cannot be regarded as united (a) by συνέχει, (b) by σύνθεσις, nor (c) by ἀφή. He now argues against (d) the view that they constitute 'a series', that they are united by τὸ ἐφεξῆς. (Cf. above, note on 971ᵃ 16.) He urges here that, whatever may be the case with some 'series', the *series of points* must be a series whose members are contiguous, since otherwise they would not form a *continuum*—i. e. they would not form a line. It appears from *Phys.* 227ᵃ 17–23 that all *continua* must have their parts 'in contact': and all things 'in contact' must be ἐφεξῆς. But there may be τὸ ἐφεξῆς without 'contact' (e. g. the numerical series), and there may be 'contact' without the contiguous plurality constituting a *continuum*.

In ll. 29–31 I read as follows :—τὸ δὲ συνεχὲς οὐδὲν ἄλλο λέγομεν ἢ τὸ ἐξ ὧν ἐστὶν ἁπτομένων· ὥστε καὶ οὕτως ἀνάγκη τὰς στιγμὰς ἅπτεσθαι, ἢ [MSS. ᾖ] εἶναι γραμμὴν συνεχῆ.

The clause τὸ δὲ συνεχὲς ... ἁπτομένων is direct, and does not depend on εἰ in l. 28. The δὲ is resumptive. καὶ οὕτως, viz. even supposing that the points are ἐφεξῆς.

ἢ εἶναι γραμμὴν συνεχῆ, viz. ᾖ ἀνάγκη ἐστὶν εἶναι κτλ.

The meaning concealed in the corrupt τὸ ἐξ ὧν ἐστὶν ἁπτομένων is rightly given by Rota, ' quod est ex se tangentibus compositum.'

[2] ᵃ1–3. The text is here hopelessly corrupt. Apelt conjectures ἔτι, εἰ ἄτοπον στιγμὴν ἐπὶ στιγμῆς εἶναι ἢ γραμμὴν καὶ ἐπὶ στιγμῆς, ἐπὶ δὲ γραμμῆς ἐπίπεδον κτλ. : and (v. *prolegg.*, p. xxii) interprets ' si fieri nequit ut puncto iuxta positum punctum adiungatur, quatenus ne linea quidem puncto iuxta posita adiungi potest neque planum lineae . . .' But I do not see how he could defend this translation of his Greek : nor do I see how 972ᵃ 3–6 connect with this opening sentence. In his German translation

contact, the line will be divided not at either of the points, but between them : whilst if they are in contact, a line will be the place of the single point.   And this is impossible.[1]

6   (j) Further, all things would be divided, i.e. be dissolved, into points ; and the point would be a part of a solid, since the solid—on the theory—consists of planes, the plane of lines, and the lines of points.   And since those constituents, of which (as their primary immanent factors) the various groups of things are composed, are 'elements', points would be 'elements' of bodies.   Hence 'elements' would be identical in nature as well as in name, and not even specifically different.[2]

12   § 5. It is clear, then, from the above arguments that a line does not consist of points.[3]

(a) But neither is it possible to subtract a point from a line. For, if a point can be subtracted, it can also be added.   But if anything is added, that to which it was added will be bigger than it was at first, if that which is added be such as to coalesce and form one whole with it.[4]   Hence a line will be bigger than another line by a point.[5]   And this is impossible.

But though it is not possible to subtract a point *as such* from a line, one may subtract it *incidentally*, viz. in so far as a point

he proposes to read ἂν ᾖ γραμμὴ καὶ ἐπὶ στιγμῇ, which he translates 'wenn auch eine Linie auf einem Punkte sein kann ': but one may envy, without wishing to imitate, this free-and-easy attitude to Greek Grammar. It seemed best to own myself defeated, and simply to print the original Greek.

[1] ᵃ3-6. For the argument, cf. above, 971ᵃ 28 ff.   But what bearing has this dilemma (εἴτε γὰρ . . . εἶθ' ἅπτονται) on the preceding lines ?

[2] ᵃ6-11. In l. 11 I read with Apelt, after the MS. Wᵃ, οὐδ' ἕτερα, for Bekker's οὐδέτερα.

The common name 'στοιχεῖον' would indicate a genuine identity of nature in the different things called ' elements': indeed, *complete* identity of nature, and not merely generic identity with specific differences.

In l. 10 ἕκαστα means, of course, not each thing, but each group or kind of thing.

[3] The writer has shown that a line is not in any sense a *sum* of points. He now shows that you cannot speak of *subtracting* a point from a line : and from this proceeds to criticize other erroneous statements about 'points '.

[4] In ᵃ15 the MSS. read τὸ προστεθὲν (τὸ προστεθήτω L) μεῖζον ἔσται τοῦ ἐξ ἀρχῆς.

Apelt conjectures τὸ ᾧ προσετέθη μεῖζον κτλ., and this seems undoubtedly right.   The corruption may have arisen from the mistaken assumption that τοῦ ἐξ ἀρχῆς means ' than the original quantum '.

[5] In ᵃ17 I read with Hayduck ἔστιι ⟨ἄρα⟩ γραμμή κτλ.

is contained in the line which one is subtracting from another line. For since, if the whole be subtracted, its beginning and its end are subtracted too ; and since the beginning and the end of a line are points : then, if it be possible to subtract a line from a line, it will be possible also thereby to subtract a point. But such a subtraction of a point is *incidental* or *per accidens*.[1]

(b) But if the limit *touches* that of which it is the limit (touches either *it* or some one of its parts), and if the point, *quâ* limit of the line, touches the line, then the line will be greater than another line by a point, and the point will consist of points. For there is nothing between two things in contact.[2]

The same argument applies in the case of division, since the 'division' is a point and, *quâ* dividing-point, is in contact with something. It applies also in the case of a solid and a plane. And the solid must consist of planes, the plane of lines, just as (on the theory) the line consists of points.[3]

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[1] ᵃ20–24. I follow Hayduck and Apelt in reading εἰ ⟨γὰρ⟩ τοῦ ὅλου ἀφαιρουμένου καὶ ἡ ἀρχὴ καὶ τὸ πέρας ἀφαιρεῖται, γραμμῆς δ᾽ ἦν ἡ ἀρχὴ καὶ τὸ πέρας στιγμή, καὶ εἰ γραμμῆς ⟨γραμμὴν⟩ ἐγχωρεῖ ἀφαιρεῖν, καὶ στιγμὴν ⟨ἂν⟩ ἐνδέχοιτο.

[2] ᵃ24–27. The writer shows that it is wrong to conceive the limit as 'in contact' with that which it limits, and the point as 'in contact' with the line or any part of it.

In l. 24 I read (with Apelt) οὗ τὸ πέρας for the MSS. οὔτε πέρας.

In l. 25 I punctuate . . . ἐκείνου τινός, ἡ δὲ στιγμή, ᾗ πέρας, γραμμῆς ἅπτεται, and in l. 26 I adopt Apelt's conjecture ἡ μὲν οὖν ⟨γραμμὴ⟩ γραμμῆς ἔσται στιγμὴ μείζων for the MSS. ᾗ μὲν οὖν γραμμῆς ἔσται στιγμὴ μείζων [N ἡ μὲν οὖν γραμμὴ ἔσται στιγμῆς μείζων].

If the point *C becomes* the limit of the line *AB*, and is therefore 'in contact' with *AB*, then (i) *BA + C* is > *BA* by the point *C*, and (ii) the terminal point *C* of the line *CAB* is the *composite* point *C + A* : for *C* and *A* are in contact whole-with-whole, and there is nothing between them.

[3] ᵃ28–30. This passage is obscure owing to its brevity. In l. 28 I read (with NWᵃ) ὁ ⟨δ᾽⟩ αὐτὸς λόγος . . ., but perhaps we ought to retain the asyndeton, in spite of its harshness. The writer's style, especially at the end of the treatise, is abrupt and compressed in the extreme. In l. 28 I read εἰ ἡ τομὴ στιγμὴ [so Zᵃ : the other MSS. read στιγμήν] καὶ, ᾗ [MSS. ἡ] τομή, ἅπτεταί τινος, and in l. 30 I accept Apelt's conjecture καὶ ⟨τὸ ἐπίπεδον⟩ ἐκ γραμμῶν.

If a line consists of points in contact, division of a line—the actual 'cut'—is itself a point, and (*quâ* dividing-point) is in contact with the adjacent points, or halves of a point, which it separates. But if so, we shall be led to the same absurdities as before (cf. 972ᵃ 24–27). Hence

30   (c) Neither [1] is it true to say of a point that it is 'the smallest constituent of a line'.

(i) For if it be called 'the smallest of the things contained in the line', what is 'smallest' is also *smaller* than those things **972ᵇ** of which it is the smallest. But in the line there is contained nothing but points and lines : and the line is not bigger than the point, for neither is the plane bigger than the line.[2] Hence the point will not be the smallest of the constituents in the line.[3]

4   (ii) And if the point is comparable in magnitude with the line, yet, since 'the smallest' involves three degrees of comparison,[4] the point will not be the *smallest* of the constituents of the line : or [5] there will be other things in the length besides

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we must not regard division as 'dividing a point', or as itself a 'point of dividing'. But if not, how can a line—which *ex hypothesi* is nothing but 'points in contact'—be 'divided'?

The writer then briefly reminds us that, if a line consists of points in contact, on the same principle a plane is a sum of lines, a solid a sum of planes, in contact with one another : and if we thus conceive solids and planes, 'the same argument' will apply to them. One plane, e.g., will be greater than another by a line, one solid greater than another by a plane, if we are able to 'subtract' a line from a plane, and a plane from a solid ; and we shall get into difficulties with 'division'.

. [1] ᵃ30 ff. We have seen that we must not predicate 'contact', 'addition', 'subtraction', or 'division' of the points in a line. In the following arguments the writer shows that we must not say of a point that it is 'the smallest constituent of a line'. No doubt he is attacking a current definition.

[2] ᵃ30–ᵇ3. The MSS. read οὐκ ἀληθὲς δὲ κατὰ στιγμὴν εἰπεῖν, οὐδ' ὅτι τὸ ἐλάχιστον [ἐλαχίστη L, καὶ ἐλαχίστη P, καὶ ἐλάχιστον Wᵃ] τῶν ἐκ γραμμῆς εἰς τὸ ἐλάχιστον [τὸ om. L] τῶν ἐνυπαρχόντων εἴρηται. τὸ δὲ ἐλάχιστον κτλ.

The reading, which I have translated, is based on suggestions of Hayduck and Apelt : but I have altered Apelt's punctuation, and substituted γ' for δέ in l. 33. I read the whole passage thus :—οὐκ ἀληθὲς δὲ κατὰ στιγμῆς εἰπεῖν, οὐδ' ὅτι τὸ ἐλάχιστον τῶν ἐν γραμμῇ. εἰ γὰρ τὸ ἐλάχιστον τῶν ἐνυπαρχόντων εἴρηται, τό γ' ἐλάχιστον, ὧν ἐστιν ἐλάχιστον, καὶ ἔλαττόν ἐστιν. ἐν δὲ τῇ γραμμῇ κτλ.

[3] ᵇ2–4. The writer assumes that *the other* constituents of the line, i.e. those presupposed in calling the point 'the smallest' constituent, are infinitesimal ('indivisible') lines : and the point is not smaller than these. The words in l. 3, οὐδὲ γὰρ αὖ τὸ ἐπίπεδον τῆς γραμμῆς, are obscure. Presumably we are to suppose that (according to the theory) just as the line consists of infinitesimal lines=points, so the plane consists of *planes-of-infinitesimal-breadth*=lines.

[4] ᵇ5. ἐν τρισὶ προσώποις. The word does not appear to be used in this sense elsewhere in Aristotle.

[5] ᵇ6. I read . . . τῶν ἐν τῇ γραμμῇ ἐλάχιστον, ⟨ἢ⟩ καὶ ἄλλ' ἄττα ἐνυπάρξει [so Hayduck for the MSS. ἐνυπάρχει] παρὰ κτλ. The insertion of ἤ seems

the points and lines, so that it will not consist of points.[1]   But, since that which is in place is either a point or a length or a plane or a solid, or some compound of these : and since the constituents of a line are in place (for the line is in place) : and since neither a solid nor a plane, nor anything compounded of these, is contained in the line :—there can be absolutely nothing in the length except points and lines.[2]

(iii) Further, since that which is called ' greater ' than that which is in place is a length or a surface or a solid : then, since the point is in place, and since that which is contained in the length besides points and lines is none of the aforementioned :—the point cannot be the smallest of the constituents of a length.[3]

(iv) Further, since ' the smallest of the things contained in a house ' is so called, without in the least comparing the house with it, and so in all other cases :—neither will the smallest of the constituents in the line be determined by comparison with

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to be required by the logic of the passage.   The writer propounds a dilemma :—

(1) If there are only two kinds of constituent in the line, one of those kinds (viz. the point) cannot be the ' smallest ' ;

(2) If, on the other hand, there are more than two kinds of constituent in the line, there must be something other than points and lines contained in it.   This he shows to be impossible in the following argument.

[1] ᵇ8. I read οὐκ ἄρ’ ἐκ στιγμῶν. If the MSS. reading (οὐ γὰρ . . .) be retained, we must translate ' For, on this supposition, it will no longer consist of points '.

In ᵇ7 τῷ μήκει is substituted for τῇ γραμμῇ. γραμμή is determinate μῆκος, ἐπιφάνεια determinate πλάτος, and σῶμα determinate βάθος, according to Arist. *Met.* 1020ᵃ 13.

[2] ᵇ8–13. In ll. 8, 9 I read with Hayduck εἰ δὲ τὸ ἐν τόπῳ ὂν ἢ στιγμὴ ἢ μῆκος [MSS. ἡ στιγμὴ μῆκος] ἢ ἐπίπεδον ἢ στερεὸν ⟨ἢ⟩ ἐκ τούτων τι . . .

[3] ᵇ13–17. In ᵇ14 I read with Hayduck ἢ ἐπιφάνεια ἢ στερεόν for the MSS. ἡ ἐπιφάνεια στερεόν.

The argument is :—The point is ' in place ', i. e. a spatial thing.   What is greater than the point, therefore, must be either a line or a plane or a solid.   Now, in a length there can be contained neither plane nor solid.   Hence there can *at most* be contained in a length one order of spatial thing (viz. line) which is greater than the point.   Hence we are *at most* entitled to apply the comparative (' smaller '), and not the superlative (' smallest '), to the point in relation to the other constituents of the line.

It is possible, I think, that we should excise εἰ in ᵇ13, and read ἔτι τοῦ ἐν τόπῳ κτλ.

the line.   Hence the term 'smallest' applied to the point will
not be suitable.[1]

21   (v) Further, that which is not in the house is not the smallest
of the constituents of the house, and so in all other cases.
Hence, since the point can exist *per se*, it will not be true to
say of it that it is 'the smallest thing in the line'.[2]

25   (d) Lastly, the point is not an 'indivisible joint'.[3]

For (i) the joint is always a limit of two things, but the
point is a limit of *one* line as well as of two.   Moreover (ii) the
point is an end, but the joint is more of the nature of a division.

Again (iii) the line and the plane will be 'joints' ⟨too⟩ : for
they are analogous to the point.   Again (iv) the joint *is* in a sense
on account of movement (which explains the verse of Empedo-
cles [4]) : but the point is found also in the immovable things.[5]

(v) Again, nobody has an infinity of joints in his body or
his hand, but he has an infinity of points.[6]   (vi) Moreover,
31   there is no joint of a stone, nor has it any : but it has points.

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[1] ^{b}17-21. In ^{b}18 I read μή τι τῆς οἰκίας συμβαλλομένης πρὸς αὐτὸ λέγεται.
The MSS. give μήτε τῆς κτλ.   Hayduck proposed μὴ τῆς, and Apelt
conjectured μήτε ⟨πρὸς τὴν οἰκίαν συμβάλλεται μήτε⟩ τῆς οἰκίας . . . .
In ^{b}21 I follow Apelt in reading ἐλάχιστον.   ἔτι εἰ for the MSS. ἐλάχιστον,
ἐπεὶ [ἐπὶ P]. . . .
The writer seems to be meeting a possible objection.   For it might be
said : ' It is mere pedantry to object to the superlative.   All we meant was
that the point is smaller than the infinitesimal lines, or at any rate than
the whole line.'

[2] ^{b}21-24. I read this passage as follows :—ἔτι εἰ τὸ μὴ ὂν ἐν τῇ οἰκίᾳ μή
ἐστι τῶν ἐν τῇ οἰκίᾳ ἐλάχιστον, ὁμοίως δὲ καὶ ἐπὶ τῶν ἄλλων, ἐνδέχεται δὲ
[so NW^{a}Z^{a}: the other MSS. read γὰρ] στιγμὴν αὐτὴν καθ' αὑτὴν εἶναι, οὐκ
ἔσται κατὰ ταύτης ἀληθὲς εἰπεῖν ὅτι τὸ ἐν γραμμῇ ἐλάχιστον. ἔτι δ' οὐκ κτλ. [So
Hayduck and Apelt : the MSS. read ἐλάχιστον, ὅτι δὲ οὐκ, or ὅτι οὐκ.]
The writer criticizes the definition on the ground that it assumes that the
point is *essentially* a constituent of a line, i. e. has no being except in a line.

[3] ^{b}25. We must not describe the point as 'an indivisible joint'.   We
do not know who thus described it, but no doubt the writer is attacking a
current description.

[4] ^{b}27-31. I read ἀνάλογον γὰρ ἔχουσιν. ἔτι [so Apelt, following W^{a}; the
other MSS. have ἔχουσιν, ὅτι] τὸ ἄρθρον διὰ φοράν [so Apelt, for the MSS.
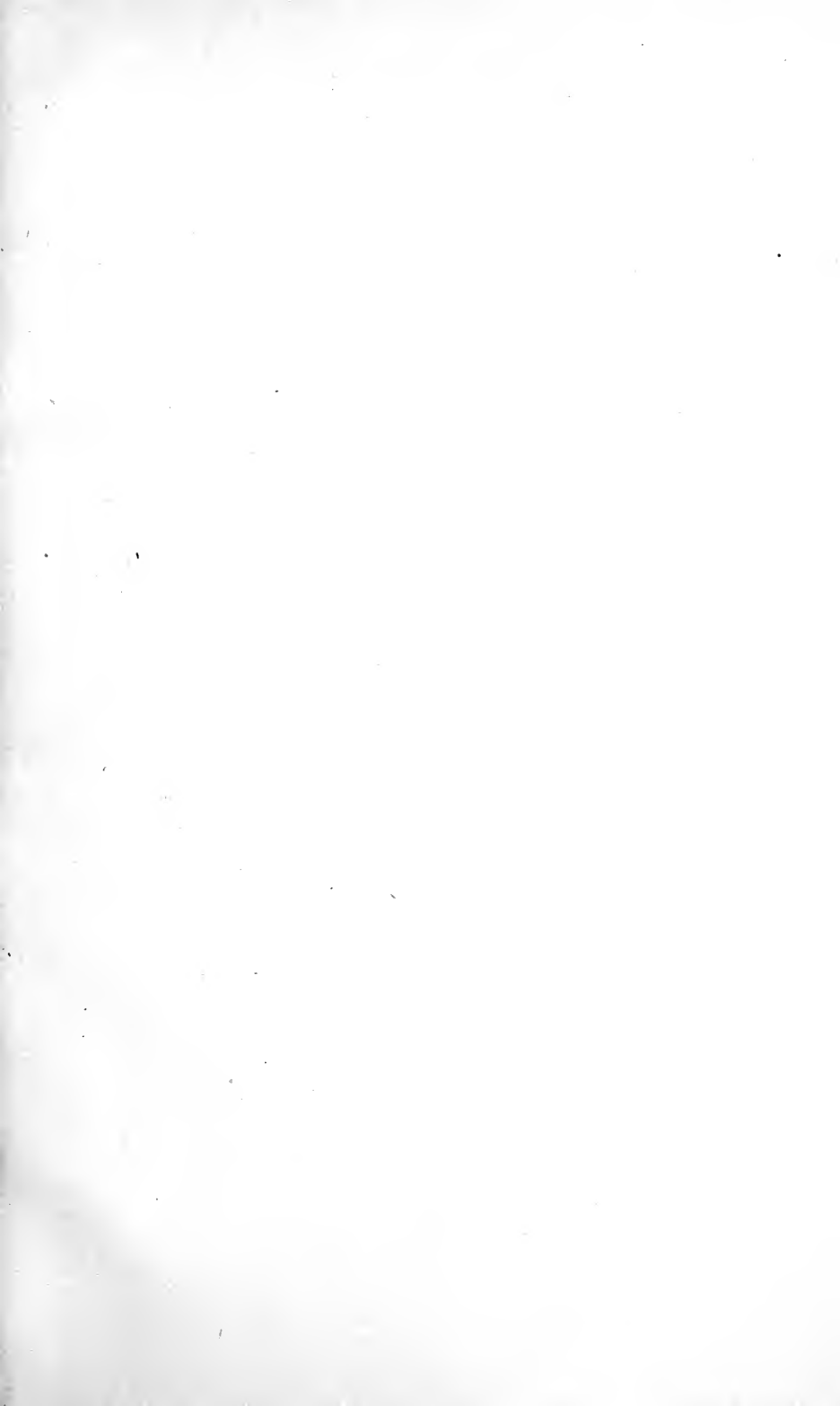διαφορά or διάφορον] πως ἐστίν . . . .
What the verse of Empedocles was, is unknown : the MSS. give ' διὸ
δεῖ ὀρθῶς', for which Diels (*Vorsokratiker*, 2nd ed., vol. 1, p. 184)
brilliantly conjectures δύω δέει ἄρθρον, 'the joint binds two'.

[5] ^{b}30. The MSS. have ἡ δὲ στιγμὴ καὶ τὸ ἐν τοῖς ἀκινήτοις. The τὸ is
unintelligible, and Hayduck is no doubt right in excising it.

[6] ^{b}31. The MS. L exhibits στόματι for σώματι in its margin.   But this
looks like a correction.   The argument is *a fortiori*.   ' In one's body—nay,
even in one's hand—there are an infinity of points. . . .'